Significance Tests Review **AP Statistics** **Name:** Key

I. Multiple Choice.

1. The average yield of a certain crop is 10.1 bushels per plant. A biologist claims that a new fertilizer will result in a greater yield when applied to the crop. A random sample of 25 of plants given the fertilizer has an average yield of 10.8 bushels and a standard deviation of 2.1 bushels. The appropriate null and alternative hypotheses to test the biologist's claim are

$\bar{x}$

(a) H_0: $\mu = \cancel{10.8}$ against H_a: $\mu > 10.8$
(b) H_0: $\mu = \cancel{10.8}$ against H_a: $\mu \neq 10.8$
C (c) H_0: $\mu = 10.1$ against H_a: $\mu > 10.1$ greater yield
(d) H_0: $\mu = 10.1$ against H_a: $\mu < 10.1$
(e) H_0: $\mu = 10.1$ against H_a: $\mu \neq 10.1$

2. A test of significance produces a P-value of 0.035. Which of the following conclusions is appropriate?

(a) Accept H_a at the $\alpha = 0.05$ level
(b) Reject H_a at the $\alpha = 0.01$ level
D (c) Fail to reject H_0 at the $\alpha = 0.05$ level
(d) Reject H_0 at the $\alpha = 0.05$ level $0.035 < 0.05$
(e) Accept H_0 at the $\alpha = 0.01$ level

3. A Type II error is

(a) rejecting the null hypothesis when it is true. Type I
(b) failing to reject the null hypothesis when it is false.
B (c) rejecting the null hypothesis when it is false. ✓
(d) failing to reject the null hypothesis when it is true. ✓
(e) more serious than a Type I error. – not necessarily

4. A researcher plans to conduct a significance test at the $\alpha = 0.05$ significance level. She designs her study to have a power of 0.85 for a particular alternative value of the parameter. The probability that the researcher will commit a Type II error for the particular alternative value of the parameter at which she computed the power is

(a) 0.05 $1 - 0.85 = 0.15$
(b) 0.15
B (c) 0.80
(d) 0.95
(e) equal to 1 – (P-value) and cannot be determined until the data have been collected.

5. In hypothesis testing β is the probability of committing a Type II error in a test with significance level α. The probability of committing a type I error is

(a) $1 - \beta$
D (b) $1 - \alpha$
(c) $\beta - \alpha$
(d) α
(e) Cannot be determined

6. The mean weight of a random sample of 35 athletes is found to be 165 ($\bar{x}$) pounds with a standard deviation of 20 pounds. It is believed that a mean weight of 160 (μ) pounds would be normal for this group. To see if there is evidence that the mean weight of the population of all athletes of this type is significantly higher than 160 pounds, the hypotheses H_0: $\mu = 160$ *vs* H_a: $\mu > 160$ are tested. You obtain a *P*-value of 0.0742. Which of the following is true?

A

(a) You have failed to obtain sufficient evidence against H_0. ← don't ever say Ho is true.
(b) At the 5% significance level, you have ~~proved that H_0 is true.~~
(c) At the 5% significance level, you have failed to prove that H_0 is true, and a larger sample size is needed to do so.
(d) Only 7.42% of the athletes weight less than 160 pounds.
(e) None of the above. A significance test is inappropriate in this setting.

7. In a one-sided hypothesis test for the mean, for a random sample of size 15 the *t*-score of the sample mean is 2.615. Is this significant at the 5 percent level? At the 1 percent level? ↳ d.f = 15-1=14

B

(a) Significant at the 1 percent level but not at the 5 percent level
(b) Significant at the 5 percent level but not at the 1 percent level
(c) Significant at both the 1 percent and 5 percent levels
(d) Significant at neither the 1 percent nor 5 percent levels
(e) Cannot be determined from the given information

Look at t-distribution chart: P-value between 0.01 & 0.02

8. You are thinking of conducting a one-sample *t* test about a population mean μ using a 0.05 significance level. Which of the following statements is correct?

E

(a) You should not carry out the test if the sample does not have a Normal distribution. ← could have t
(b) You can safely carry out the test if there are no outliers, regardless of the sample size. ← also look for strong skewness
(c) You can carry out the test if a graph of the data shows no strong skewness, regardless of the sample size. outliers?
(d) You can carry out the test only if the population standard deviation is known. ← z-test
(e) You can safely carry out the test if your sample size is at least 30.

9. A random sample of 100 likely voters in a small city produced 59 voters in favor of Candidate A. The observed value of the test statistic for testing H_0: $p = 0.5$ *vs* H_a: $p > 0.5$ is

B

(a) $z = \dfrac{0.59-0.5}{\sqrt{\dfrac{\cancel{0.59(0.41)}}{100}}}$

(b) $z = \dfrac{0.59-0.5}{\sqrt{\dfrac{0.5(0.5)}{100}}}$

(c) $z = \dfrac{\cancel{0.5-0.59}}{\sqrt{\dfrac{0.59(0.41)}{100}}}$

(d) $z = \dfrac{\cancel{0.5-0.59}}{\sqrt{\dfrac{0.5(0.5)}{100}}}$

(e) $\cancel{t} = \dfrac{0.59-0.5}{\sqrt{\dfrac{0.5(0.5)}{100}}}$

10. An SRS of 100 postal employees found that the average time these employees had worked at the postal service was 7 years with standard deviation 2 years. Do these data provide convincing evidence the population of postal employees has changed from the value of 7.5 that was true 20 years ago? To determine this, we test the hypotheses H_0: $\mu = 7.5$ *vs* H_a: $\mu \neq 7.5$ using a one-sample *t* test. What conclusion should we draw at the 5% significance level?

$t = \dfrac{7-7.5}{2/\sqrt{100}} \approx -2.5$ P-value = 0.01406 ← small so reject Ho

A

(a) There is convincing evidence that the mean time working with the postal service has changed.
(b) There is not convincing evidence that the mean time working with the postal service has change.
(c) There is convincing evidence that the mean time working with the postal service is still 7.5 years.
(d) There is convincing evidence that the mean time working with the postal service is now 7 years.
(e) We cannot draw a conclusion at the 5 % significance level. The sample size is too small.

Multiple Choice Answers: 1. C, 2. D, 3. B, 4. B, 5. D, 6. A, 7. B, 8. E, 9. B, 10. A

For questions 11 & 12, define the parameter of interest in context, and state appropriate hypotheses for performing a significance test.

11. The diameter of a spindle in a small motor is supposed to be 5 mm. If the spindle is either too small or too large, the motor will not work properly. The manufacturer measures the diameter in a sample of motors to determine whether the mean diameter has moved away from the target.

$H_0: \mu = 5$ mm
$H_a: \mu \neq 5$mm

μ = mean diameter of a spindle in a small motor

12. Mr. Starnes believes that less than 75% of the students at his school completed their math homework last night. The math teachers inspect the homework assignments form a random sample of students at the school to help Mr. Starnes test his claim.

$H_0: p = 0.75$
$H_a: p < 0.75$

p = proportion of students at his high school who completed their math HW last night

YOU WILL NEED TO USE YOUR OWN PAPER FOR THE REMAINDER OF THE REVIEW.

For the following questions, make sure to show all of your work and indicate clearly the methods you use.

13. A software company is trying to decide whether to produce an upgrade of one of tis programs. Customers would have to pay \$100 for the upgrade. For the upgrade to be profitable, the company needs to sell it to more than 20% of their customers. You contact a random sample of 60 customers and find that 16 would be willing to pay \$100 for the upgrade.

(a) Do the sample data give good evidence that more than 20% of the company's customers are willing to purchase the upgrade? Carry out an appropriate test at the $\alpha = 0.05$ significance level.

(b) Which would be a more serious mistake in this setting – a Type I error or a Type II error? Justify your answer.

(c) Describe two ways to increase the power of the test in part (a).

14. Bottles of a popular cola are supposed to contain 300 milliliters of cola. There is some variation from bottle to bottle because the filling machinery is not perfectly precise. An inspector measures the contents of six randomly selected bottles from a single day's production. The results are

299.4	297.7	301.0	298.9	300.2	297.0

(a) Do these data provide convincing evidence that the mean amount of cola in all bottles filled that day differs from the target value of 300 ml?

(b) Construct and interpret a 95% confidence interval for the population mean μ. What additional information does the confidence interval provide?

15. "I can't get through my day without coffee" is a common statement from many students. Assumed benefits include keeping students awake during lectures and making them more alert for exams and tests. Students in a statistics class designed an experiment to measure memory retention with and without drinking a cup of coffee one hour before a test. This experiment took place on two different days in the same week (Monday and Wednesday). Ten students were used. Each student received no coffee or one cup of coffee one hour before the test on a particular day. The test consisted of a series of words flashed on a screen, after which the student had to write down as many of the words as possible. On the other day, each student received a different amount of coffee (none or one cup).

(a) One of the researchers suggested that all the subjects in the experiment drink no coffee before Monday's test and one cup of coffee before Wednesday's test. Explain to the researcher why this is a bad idea and suggest a better method of deciding when each subject receives the two treatments.

(b) The data from the experiment are provided in the table below. Set up and carry out an appropriate test to determine whether there is convincing evidence that drinking coffee improves memory.

One Cup − No Cup → improvement

Student	No Cup	One Cup	Difference
1	24	25	+1
2	30	31	+1
3	22	23	+1
4	24	24	0
5	26	27	+1
6	23	25	+2
7	26	28	+2
8	20	20	0
9	27	27	0
10	28	30	+2

16. A government report says that the average amount of money spent per U.S. household per week on food is about \$158. A random sample of 50 households in a small city is selected, and their weekly spending on food is recorded. The sample data have a mean of \$165 and a standard deviation of \$20. Is there convincing evidence that the mean weekly spending on food in this city differs from the national figure of \$158?

Need more than

13. $p = 0.20 \leftarrow$ $n = 60$ $\hat{p} = \frac{16}{60} \approx 0.267$

(a) $\boxed{\text{State:}}$ We want to perform a test of:

$H_0: p = 0.20$

$H_a: p > 0.20$

where p = proportion of customers willing to purchase the upgrade. We will use $\alpha = 0.05$.

$\boxed{\text{Plan:}}$ One-sample z test for p

1. Random sample of 60 customers
   10%: $60 \leq \frac{1}{10}$(all customers) Reasonable to assume
2. Large Counts: $60(0.2) = 12 \geq 10$ safe to use Normal distribution
   $60(0.8) = 48 \geq 10$

$\boxed{\text{Do:}}$ $z = \frac{0.267 - 0.2}{\sqrt{\frac{0.2(0.8)}{60}}} \approx 1.29$ P-value ≈ 0.0984

$\boxed{\text{Conclude:}}$ Because the P-value of 0.0984 is greater than $\alpha = 0.5$, we fail to reject H_0. We do not have convincing evidence that the proportion of customers willing to purchase the upgrade is more than 0.20.

(b) A Type I Error would be more serious because the company would think they would have the support for the upgrade when they really don't, causing the company to lose money.

(c) You could increase α or the sample size n.

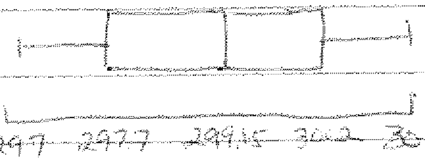
14. (a) $\boxed{\text{State:}}$ We want to perform a test of:

$$H_0: \mu = 300 \text{ mL}$$
$$H_a: \mu \neq 300 \text{ mL}$$

where μ = mean amount of cola in all bottles filled that day. Because an α has not been stated, we'll use $\alpha = 0.05$.

$\boxed{\text{Plan:}}$ One-sample t-test for μ

1. Six randomly selected bottles
   10%: $6 \leq 10$(all bottles filled that day)
   Reasonable to assume.
2. $6 < 30 \rightarrow$ graph data:



   Graph shows no outliers or strong skewness. Safe to use t-procedures.

$\boxed{\text{Do:}}$ $\bar{x} = 299.03$ $s \approx 1.502$ $df = 6-1 = 5$

$t = \dfrac{299.03 - 300}{1.502/\sqrt{6}} \approx -1.576$ P-value ≈ 0.1760

$\boxed{\text{Conclude:}}$ Because the P-value of 0.1760 is larger than $\alpha = 0.05$, we fail to reject H_0. We do not have convincing evidence that the mean amount of cola in all bottles filled that day differs from 300 mL.

(b) on next page↓

14. (b) Conditions met (checked in part (a))

95% Confidence Interval:

$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 299.03 \pm 2.571 \frac{1.502}{\sqrt{6}}$

$= (297.46, 300.61)$

For t^* → use $df = 6-1 = 5$ and 95% CL

We are 95% confident the interval from 297.46 mL to 300.61 mL captures the true mean amount of cola in all bottles filled that day.

The confidence interval not only tells us that we will fail to reject H_0 at the 5% significance level ($1-0.95=0.05$), it also gives us plausible values for the mean at a 95% confidence level.

15. (a) By making all of our subjects have no coffee on the first day and one cup of the second day doesn't allow us to see if the variable of coffee improves the scores because of outside factor. For example, students may have slept better before Monday's test because of the weekend.

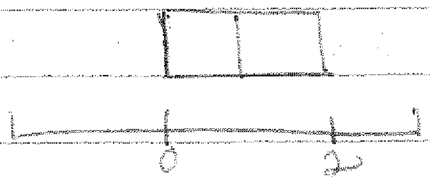
(b) $\boxed{\text{State:}}$ We want to perform a test of:

$$H_0: \mu_D = 0$$
$$H_a: \mu_D > 0$$

where μ_D is the mean difference in score (One cup − No cup). Since it is not stated, we'll use $\alpha = 0.05$.

$\boxed{\text{Plan:}}$ Matched pairs t test

1. Randomized order of treatments (subjects do not need to be randomly selected here so we don't have to check 10%)
2. Graph data → differences →



Graph does not show any outliers or strong skewness.
Safe to use t-procedures

$\boxed{\text{Do:}}$ $\bar{x} \approx 1$ $\quad s = 0.8165$ $\quad$ d.f. $= 10 - 1 = 9$

$t = \dfrac{1 - 0}{0.8165/\sqrt{10}} \approx 3.87$ $\quad$ P-value ≈ 0.0019

$\boxed{\text{Conclude:}}$ Because the P-value of 0.0019 is less than $\alpha = 0.05$, we reject H_0. We have convincing evidence that having one cup of coffee does improve memory

16. $n = 50$ $\bar{x} = 165$ $s = 20$

$\boxed{\text{State:}}$ We want to perform a test of

$$H_0: \mu = 158$$
$$H_a: \mu \neq 158$$

where μ = the mean weekly spending (in dollars) on food in this city. We will use $\alpha = 0.05$.

$\boxed{\text{Plan:}}$ One-sample t-test for μ

1. Random sample of 50 households
   10%: $50 \leq \frac{1}{10}$(all households in city)
   Reasonable to assume
2. $50 \geq 30$. By CLT, it is okay to use t-procedures.

$\boxed{\text{Do:}}$ d.f. $= 50 - 1 = 49$

$$t = \frac{165 - 158}{20/\sqrt{50}} \approx 0.707 \quad \text{P-value} \approx 0.4828$$

$\boxed{\text{Conclude:}}$ Because the P-value of 0.4828 is much greater than $\alpha = 0.05$, we fail to reject H_0. We do not have convincing evidence to say the mean weekly spending on food for this city differs from the national figure of \$158.