1. (a) Recognizing an AP (M1)

$u_1 = 15$ $d = 2$ $n = 20$ (A1) 4

substituting into $u_{20} = 15 + (20 - 1) \times 2$ M1

= 53 (that is, 53 seats in the 20th row) A1

(b) Substituting into $S_{20} = \frac{20}{2}(2(15) + (20-1)2)$ (or into $\frac{20}{2}(15 + 53)$) M1

= 680 (that is, 680 seats in total) A1 2

[6]

2. (a) $5000(1.063)^n$ A1 1

(b) Value = $\$5000(1.063)^5$ (= \$6786.3511...)

= \$6790 to 3 sf (Accept \$6786, or \$6786.35) A1 1

(c) (i) $5000(1.063)^n > 10000$ or $(1.063)^n > 2$ A1 1

(ii) Attempting to solve the inequality «log (1.063) > log 2 (M1)

$n > 11.345...$ (A1)

12 years A1 3

***Note**: Candidates are likely to use TABLE or LIST on a GDC to find n. A good way of communicating this is suggested below.*

Let $y = 1.063^x$ (M1)

When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)

$x = 12$ *ie* 12 years A1 3

[6]

3. (a) $u_1 = S_1 = 7$ (A1) (C1)

(b) $u_2 = S_2 - u_1 = 18 - 7$

$= 11$ (A1)

$d = 11 - 7$ (M1)

$= 4$ (A1) (C3)

(c) $u_4 = u_1 + (n-1)d = 7 + 3(4)$ (M1)

$u_4 = 19$ (A1) (C2)

[6]

4. For using $u_3 = u_1 r^2 = 8$ (M1)

$8 = 18r^2$ (A1)

$r^2 = \frac{8}{18}\left(=\frac{4}{9}\right)$

$r = \pm\frac{2}{3}$ (A1)(A1)

$S_\infty = \frac{u_1}{1-r},$

$S_\infty = 54, \frac{54}{5} (=10.8)$ (A1)(A1)(C3)(C3)

[6]

5. (a) (i) Neither

(ii) Geometric series

(iii) Arithmetic series

(iv) Neither (C3)

Note: *Award (A1) for geometric correct, (A1) for arithmetic correct and (A1) for* ***both*** *"neither". These may be implied by blanks* ***only*** *if GP* ***and*** *AP correct.*

(b) (Series (ii) is a GP with a sum to infinity)

Common ratio $\frac{3}{4}$ (A1)

$S_\infty = \frac{a}{1-r}\left(=\frac{1}{1-\frac{3}{4}}\right)$ (M1)

$= 4$ (A1) (C3)

Note: *Do* ***not*** *allow* ***ft*** *from an incorrect series.*

[6]

6. (a) (i) \$11400, \$11800 (A1) 1

(ii) Total salary $= \frac{10}{2}(2\times 11\,000 + 9\times 400)$ (A1)

$= \$128000$

(A1) (N2) 2

(b) (i) \$10700, \$11449 (A1)(A1)

(ii) 10^{th} year salary $= 10\,000(1.07)^9$ (A1)

$= \$18384.59$ or \$18400 or \$18385 (A1)(N2) 4

(c) **EITHER**

Scheme A $S_A = \frac{n}{2}\left(2 \times 11\,000 + (n-1)400\right)$ (A1)

Scheme B $S_B = \frac{10\,000(1.07^n - 1)}{1.07 - 1}$ (A1)

Solving $S_B > S_A$ (accept $S_B = S_A$, giving $n = 6.33$) (may be implied)(M1)

Minimum value of n is 7 years.
(A1) (N2)

OR

Using trial and error (M1)

	Arturo	Bill
6 years	\$72 000	\$71532.91
7 years	\$85 400	\$86 540.21

(A1)(A1)

Note: *Award (A1) for **both** values for 6 years, and (A1) for **both** values for 7 years.*

Therefore, minimum number of years is 7. (A1) (N2) 4

[11]

7. Arithmetic sequence $d = 3$ (may be implied) (M1)(A1)
$n = 1250$ (A2)

$S = \frac{1250}{2}(3 + 3750)$ $\left(\text{or } S = \frac{1250}{2}(6 + 1249 \times 3)\right)$ (M1)

$= 2\,345\,625$ (A1) (C6)

[6]

8. Arithmetic sequence (M1)
$a = 200$ $d = 30$ (A1)

(a) Distance in final week = $200 + 51 \times 30$ (M1)
$= 1730$ m (A1) (C3)

(b) Total distance = $\frac{52}{2}$ [2.200 + 51.30] (M1)

= 50180 m (A1) (C3)

Note: *Penalize once for absence of units ie award A0 the first time units are omitted, A1 the next time.*

[6]

9. (a) (i) Area B = $\frac{1}{16}$, area C = $\frac{1}{64}$ (A1)(A1)

(ii) $\frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ $\frac{\frac{1}{64}}{\frac{1}{16}} = \frac{1}{4}$ (Ratio is the same.) (M1)(R1)

(iii) Common ratio = $\frac{1}{4}$ (A1) 5

(b) (i) Total area (S_2) = $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ = (= 0.3125) (0.313, 3 sf) (A1)

(ii) Required area = S_8 = $\frac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^8\right)}{1-\frac{1}{4}}$ (M1)

= 0.333328 2(471...) (A1)

= 0.333328 (6 sf) (A1) 4

Note: *Accept result of adding together eight areas correctly.*

(c) Sum to infinity = $\frac{\frac{1}{4}}{1-\frac{1}{4}}$ (A1)

= $\frac{1}{3}$ (A1) 2

[11]

10. (a) $u_4 = u_1 + 3d$ or $16 = -2 + 3d$ (M1)

$d = \frac{16-(-2)}{3}$ (M1)

$= 6$ (A1) (C3)

(b) $u_n = u_1 + (n-1)6$ or $11998 = -2 + (n-1)6$ (M1)

$n = \frac{11998+2}{6} + 1$ (A1)

$= 2001$ (A1) (C3)

[6]

11. (a) Ashley

AP 12 + 14 + 16 + ... to 15 terms (M1)

$S_{15} = \frac{15}{2}[2(12) + 14(2)]$ (M1)

$= 15 \times 26$

$= 390$ hours (A1) 3

(b) Billie

GP $12, 12(1.1), 12(1.1)^2\ldots$ (M1)

(i) In week 3, $12(1.1)^2$ (A1)

$= 14.52$ hours (AG)

(ii) $S_{15} = \frac{12[(1.1)^{15} - 1]}{1.1 - 1}$ (M1)

$= 381$ hours (3 sf) (A1) 4

(c) $12\,(1.1)^{n-1} > 50$ (M1)

$(1.1)^{n-1} > \frac{50}{12}$ (A1)

$(n-1)\ln 1.1 > \ln\frac{50}{12}$

$n - 1 > \frac{\ln\frac{50}{12}}{\ln 1.1}$ (A1)

$n - 1 > 14.97$

$n > 15.97$

$\Rightarrow$ Week 16 (A1)

OR

$12(1.1)^{n-1} > 50$ (M1)

By trial and error

$12(1.1)^{14} = 45.6$, $12(1.1)^{15} = 50.1$ (A1)

$\Rightarrow n - 1 = 15$ (A1)

$\Rightarrow n = 16$ (Week 16) (A1) 4

[11]

12. (a) $u_1 = 7$, $d = 2.5$ (M1)

$u_{41} = u_1 + (n-1)d = 7 + (41-1)2.5$

$= 107$ (A1) (C2)

(b) $S_{101} = \frac{n}{2}[2u_1 + (n-1)d]$

$= \frac{101}{2}[2(7) + (101-1)2.5]$ (M1)

$= \frac{101(264)}{2}$

$= 13332$ (A1) (C2)

[4]

13. (a) $r = \frac{360}{240} = \frac{240}{160} = \frac{3}{2} = 1.5$ (A1) 1

(b) 2002 is the 13th year. (M1)

$u_{13} = 160(1.5)^{13-1}$ (M1)

$= 20759$ (Accept 20760 or 20800.) (A1) 3

(c) $5000 = 160(1.5)^{n-1}$

$\frac{5000}{160} = (1.5)^{n-1}$ (M1)

$\log\left(\frac{5000}{160}\right) = (n-1)\log 1.5$ (M1)

$n - 1 = \frac{\log\left(\frac{5000}{160}\right)}{\log 1.5} = 8.49$ (A1)

$\Rightarrow n = 9.49 \Rightarrow 10^{\text{th}}$ year

$\Rightarrow 1999$ (A1)

OR

Using a gdc with $u_1 = 160$, $u_{k+1} = \frac{3}{2}u_k$, $u_9 = 4100$, $u_{10} = 6150$ (M2)

1999 (G2) 4

(d) $S_{13} = 160\left[\frac{1.5^{13} - 1}{1.5 - 1}\right]$ (M1)

= 61958 (Accept 61960 or 62000.) (A1) 2

(e) Nearly everyone would have bought a portable telephone so there would be fewer people left wanting to buy one. (R1)

OR

Sales would saturate. (R1) 1

[11]

14. (a) $a_1 = 1000$, $a_n = 1000 + (n - 1)250 = 10000$ (M1)

$n = \frac{10\,000 - 1000}{250} + 1 = 37.$

She runs 10 km on the 37th day. (A1)

(b) $S_{37} = \frac{37}{2}(1000 + 10000)$ (M1)

She has run a total of 203.5 km (A1)

[4]

15. $a = 5$

$a + 3d = 40$ (may be implied) (M1)

$d = \frac{35}{3}$ (A1)

$T_2 = 5 + \frac{35}{3}$ (A1)

$= 16\frac{2}{3}$ or $\frac{50}{3}$ or 16.7 (3 sf) (A1) (C4)

[4]

16. $S = \frac{u_1}{1-r} = \frac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)}$ (M1)(A1)

$= \frac{2}{3} \times \frac{3}{5}$ (A1)

$= \frac{2}{5}$ (A1) (C4)

[4]

17. (a) Plan A: 1000, 1080, 1160... Plan B: 1000, 1000(1.06), $1000(1.06)^2$…

2nd month: \$1060, 3rd month: \$1123.60 (A1)(A1) 2

(b) For Plan A, $T_{12} = a + 11d$

$= 1000 + 11(80)$ (M1)

$= \$1880$ (A1)

For Plan B, $T_{12} = 1000(1.06)^{11}$ (M1)

$= \$1898$ (to the nearest dollar) (A1) 4

(c) (i) For Plan A, $S_{12} = \frac{12}{2}[2000 + 11(80)]$ (M1)

$= 6(2880)$

$= \$17280$ (to the nearest dollar) (A1)

(ii) For Plan B, $S_{12} = \frac{1000(1.06^{12} - 1)}{1.06 - 1}$ (M1)

$= \$16870$ (to the nearest dollar) (A1) 4

[10]

18. (a) $\$1000 \times 1.075^{10} = \$2061$ (nearest dollar) (A1) (C1)

(b) $1000(1.075^{10} + 1.075^9 + \ldots + 1.075)$ (M1)

$= \dfrac{1000(1.075)(1.075^{10} - 1)}{1.075 - 1}$ (M1)

= \$15208 (nearest dollar) (A1) (C3)

[4]

19. $17 + 27 + 37 + \ldots + 417$

$17 + (n - 1)10 = 417$ (M1)

$10(n - 1) = 400$

$n = 41$ (A1)

$S_{41} = \dfrac{41}{2}(2(17) + 40(10))$ (M1)

$= 41(17 + 200)$

$= 8897$ (A1)

OR

$S_{41} = \dfrac{41}{2}(17 + 417)$ (M1)

$= \dfrac{41}{2}(434)$

$= 8897$ (A1) (C4)

[4]

20. $S_5 = \dfrac{5}{2}\{2 + 32\}$ (M1)(A1)(A1)

$S_5 = 85$ (A1)

OR

$a = 2,\ a + 4d = 32$ (M1)

$\Rightarrow 4d = 30$

$d = 7.5$ (A1)

$S_5 = \dfrac{5}{2}(4 + 4(7.5))$ (M1)

$= \dfrac{5}{2}(4 + 30)$

$S_5 = 85$ (A1) (C4)

[4]