1. **METHOD 1**

$9 = 3^2,\ 27 = 3^3$ (A1)(A1)

expressing as a power of 3, $(3^2)^{2x} = (3^3)^{1-x}$ (M1)

$3^{4x} = 3^{3-3x}$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

METHOD 2

$2x \log 9 = (1 - x) \log 27$ (M1)(A1)(A1)

$\frac{2x}{1-x} = \frac{\log 27}{\log 9} \left(= \frac{3}{2} \right)$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

Notes: *Candidates may use a graphical method.*
Award (M1)(A1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 0.4285…., and (A1) for $\frac{3}{7}$.

[6]

2. (a) $\log_3 x - \log_3 (x-5) = \log_3 \left(\frac{x}{x-5} \right)$ (A1)

$A = \frac{x}{x-5}$ (A1) (C2)

Note: *If candidates have an incorrect or no answer to part (a) award (A1)(A0)*
if $\log \left(\frac{x}{x-5} \right)$ *seen in part (b).*

(b) **EITHER**

$\log_3\left(\frac{x}{x-5}\right)=1$

$\frac{x}{x-5}=3^1\left(=3\right)$ (M1)(A1)(A1)

$x=3x-15$

$-2x=-15$

$x=\frac{15}{2}$ (A1) (C4)

OR

$\frac{\log_{10}\left(\frac{x}{x-5}\right)}{\log_{10}3}=1$ (M1)(A1)

$\log_{10}\left(\frac{x}{x-5}\right)=\log_{10}3$ (A1)

$x=7.5$ (A1) (C4)

[6]

3. METHOD 1

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right)=\log_{10}x-\log_{10}y^2-\log_{10}\sqrt{z}$ (A1)(A1)(A1)

$\log_{10}y^2=2\log_{10}y$ (A1)

$\log_{10}\sqrt{z}=\frac{1}{2}\log z$ (A1)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right)=\log_{10}x-2\log y-\frac{1}{2}\log z$

$=p-2q-\frac{1}{2}r$ (A1) (C2)(C2)(C2)

METHOD 2

$x = 10, y^2 = 10^{2p}, \ \sqrt{z} = 10^{\frac{r}{2}}$ (A1)(A1)(A1)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10}\left(\frac{10^p}{10^{2q}10^{\frac{r}{2}}}\right)$ (A1)

$= \log_{10}\left(10^{p-2q-\frac{r}{2}}\right)\left(= p - 2q - \frac{r}{2}\right)$ (A2) (C2)(C2)(C2)

[6]

4. **METHOD 1**

$\log x^2 = 2\log x$ (A1)

$\log\sqrt{y} = \frac{1}{2}\log y$ (A1)

$\log z^3 = 3\log z$ (A1)

$2\log x + \frac{1}{2}\log y - 3\log z$ (A1)(A1)

$2a + \frac{1}{2}b - 3c$ (A1) (C6)

METHOD 2

$x^2 = 10^{2a}, \ \sqrt{y} = 10^{\frac{b}{2}}, \ z^3 = 10^{3c}$ (A1)(A1)(A1)

$\log_{10}\left(\frac{x^2\sqrt{y}}{z^3}\right) = \log_{10}\left(\frac{10^{2a}\times 10^{\frac{b}{2}}}{10^{3c}}\right)$ (A1)

$= \log_{10}\left(10^{2a+\frac{b}{2}-3c}\right) \ \left(= 2a + \frac{b}{2} - 3c\right)$ (A2)

[6]

5. (a) $\log_5 x^2 = 2\log_5 x$ (M1)

$= 2y$ (A1) (C2)

(b) $\log_5 \frac{1}{x} = -\log_5 x$ (M1)

$= -y$ (A1) (C2)

(c) $\log_{25} x = \frac{\log_5 x}{\log_5 25}$ (M1)

$= \frac{1}{2} y$ (A1) (C2)

[6]

6. $\log_{27}(x(x - 0.4)) = 1$ (M1)(A1)

$x^2 - 0.4x = 27$ (M1)

$x = 5.4$ or $x = -5$ (G2)

$x = 5.4$ (A1) (C6)

Note: *Award (C5) for giving both roots.*

[6]

7. **METHOD 1**

$\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = 2 - 1 + \frac{1}{2}$ (M1)

$\Rightarrow \frac{3}{2} = \log_9 x$ (A1)

$\Rightarrow x = 9^{\frac{3}{2}}$ (M1)

$\Rightarrow x = 27$ (A1) (C4)

METHOD 2

$\log 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\frac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

8. $\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)$ (M1)

$2\log_{10}\left(\frac{P}{QR^3}\right) = 2(\log_{10}P - \log_{10}(QR^3))$ (M1)

$= 2(1\log_{10}P - \log_{10}Q - 3\log_{10}R)$ (M1)

$= 2(x - y - 3z)$

$= 2x - 2y - 6z$ or $2(x - y - 3z)$ (A1)

[4]

9. (a) $\log_2 5 = \frac{\log_a 5}{\log_a 2}$ (M1)

$= \frac{y}{x}$ (A1) (C2)

(b) $\log_a 20 = \log_a 4 + \log_a 5$ or $\log_a 2 + \log_a 10$ (M1)

$= 2\log_a 2 + \log_a 5$

$= 2x + y$ (A1) (C2)

[4]

10. $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

$3^{2x-2} = 3^{-2x}$ (M1) (A1)

$2x - 2 = -2x$ (A1)

$x = \frac{1}{2}$ (A1) (C4)

[4]

11. $4^{3x-1} = 1.5625 \times 10^{-2}$

$(3x - 1)\log_{10} 4 = \log_{10} 1.5625 - 2$ (M1)

$\Rightarrow 3x - 1 = \frac{\log_{10} 1.5625 - 2}{\log_{10} 4}$ (A1)

$\Rightarrow 3x - 1 = -3$ (A1)

$\Rightarrow x = -\frac{2}{3}$ (A1) (C4)

[4]