**Solutions Unit 3 - Exponentials**

**1a.** *[2 marks]*

The number of bacteria in two colonies,  and , starts increasing at the same time.

The number of bacteria in colony  after  hours is modelled by the function .

Find the number of bacteria in colony  initially.

## Markscheme

correct substitution into formula ***(A1)***

*eg* 

 bacteria in the dish ***A1 N2***

***[2 marks]***

**1b.** *[3 marks]*

Find the number of bacteria in colony  after four hours.

## Markscheme

correct substitution into formula ***(A1)***

*eg* 

 ***(A1)***

 bacteria in the dish (integer answer only) ***A1 N3***

***[3 marks]***

**1c.** *[3 marks]*

How long does it take for the number of bacteria in colony  to reach ?

## Markscheme

correct equation ***(A1)***

*eg* 

valid attempt to solve ***(M1)***

*eg* graph, use of logs



 (hours) ***A1 N3***

***[3 marks]***

**1d.** *[3 marks]*

The number of bacteria in colony  after  hours is modelled by the function .

After four hours, there are  bacteria in colony . Find the value of .

## Markscheme

valid attempt to solve ***(M1)***

*eg* , use of logs

correct working ***(A1)***

*eg* sketch of intersection, 



 (exact),  ***A1 N3***

***[3 marks]***

**1e.** *[4 marks]*

The number of bacteria in colony  after  hours is modelled by the function .

The number of bacteria in colony  first exceeds the number of bacteria in colony  after  hours, where . Find the value of .

## Markscheme

**METHOD 1**

setting up an equation or inequality (accept any variable for ) ***(M1)***

*eg* 

correct working ***(A1)***

*eg* sketch of intersection, 

 (accept ) ***(A1)***

 (integer answer only) ***A1 N3***

**METHOD 2**

 (from earlier work)

 ***A1A1***

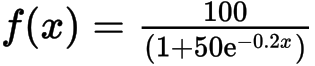
valid reasoning ***(R1)***

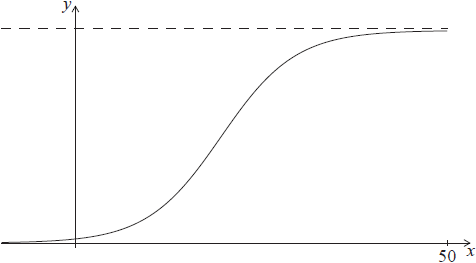
*eg*  **and** 

 (integer answer only) ***A1 N3***

***[4 marks]***

**2a.** *[1 mark]*

Let  . Part of the graph of  is shown below.



Write down  .

## Markscheme

 (exact),  ***A1 N1***

***[1 mark]***

**2b.** *[2 marks]*

Solve  .

## Markscheme

setting up equation ***(M1)***

*eg*   , sketch of graph with horizontal line at 

 ***A1 N2***

***[2 marks]***

**2c.** *[3 marks]*

Find the range of  .

## Markscheme

upper bound of  is  ***(A1)***

lower bound of  is  ***(A1)***

range is  ***A1 N3***

***[3 marks]***

**3a.** *[1 mark]*

Jose takes medication. After *t* minutes, the concentration of medication left in his bloodstream is given by  , where *A* is in milligrams per litre.

Write down  .

## Markscheme

 ***A1 N1***

***[1 mark]***

**3b.** *[2 marks]*

Find the concentration of medication left in his bloodstream after 50 minutes.

## Markscheme

substitution into formula ***(A1)***

e.g.  , 

 ***A1 N2***

***[2 marks]***

**3c.** *[5 marks]*

At 13:00, when there is no medication in Jose’s bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again?

## Markscheme

set up equation ***(M1)***

e.g. 

attempting to solve  ***(M1)***

e.g. graph, use of logs

correct working ***(A1)***

e.g. sketch of intersection, 

 ***A1***

correct time 18:33 or 18:34 (accept 6:33 or 6:34 but nothing else) ***A1 N3***

***[5 marks]***

**4.** *[3 marks]*

Let  , for  .

(i) Write down the *x*-coordinate of the maximum point on the graph of *f* .

(ii) Write down the interval where *f* is increasing.

## Markscheme

(i)  ***A1 N1***

(ii) correct interval, with right end point  ***A1A1 N2***

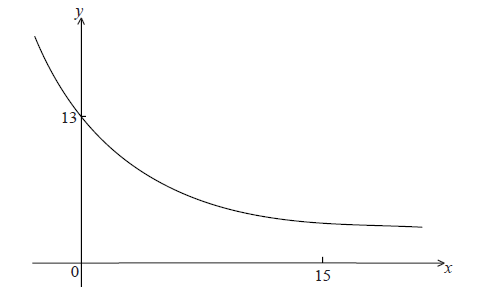
e.g.  , 

**Note**: Accept any inequalities in the right direction.

***[3 marks]***

**5a.** *[2 marks]*

Let  . Part of the graph of *f* is shown below.



The *y*-intercept is at (0, 13) .

Show that  .

## Markscheme

substituting (0, 13) into function ***M1***

e.g. 

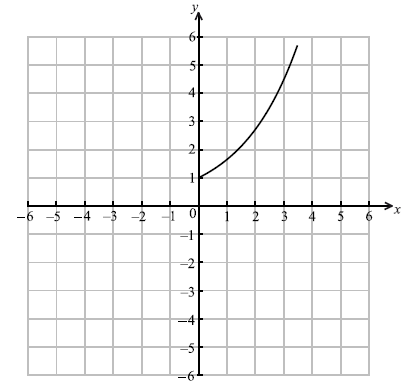
 ***A1***

 ***AG N0***

***[2 marks]***

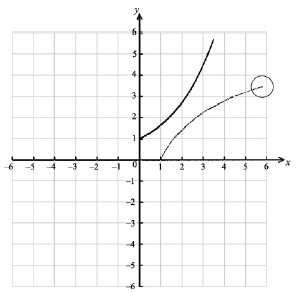
**6a.** *[3 marks]*

Let *f* be the function given by  ,  . The diagram shows the graph of *f* .



On the same diagram, sketch the graph of  .

## Markscheme

 ***A1A1A1 N3***

**Note**: Award ***A1*** for approximately correct (reflected) shape, ***A1*** for right end point in circle, ***A1*** for through  .

**6b.** *[1 mark]*

Write down the range of  .

## Markscheme

 ***A1 N1***

***[1 mark]***

**6c.** *[3 marks]*

Find  .

## Markscheme

interchanging *x* and *y* (seen anywhere) ***M1***

e.g. 

evidence of changing to log form ***A1***

e.g.  ,  (any base),  (any base)

 ***A1 N1***

***[3 marks]***

**8a.** *[3 marks]*

Let  .

(i) Show that  .

(ii) Write down the domain of  .

## Markscheme

(i) interchanging *x* and *y* (seen anywhere) ***M1***

e.g. 

correct manipulation ***A1***

e.g.  , 

 ***AG N0***

(ii)  ***A1 N1***

***[3 marks]***

**9a.** *[2 marks]*

The number of bacteria, *n* , in a dish, after *t* minutes is given by  .

Find the value of *n* when  .

## Markscheme

 ***(A1)***

 ***A1 N2***

***[2 marks]***

**11a.** *[6 marks]*

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After *n* years the number of taxis, *T*, in the city is given by



(i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

## Markscheme

(i)  ***(A1)***



 ***A1 N2***

(ii) evidence of doubling ***(A1)***

e.g. 560

setting up equation ***A1***

e.g. , 

  ***(A1)***

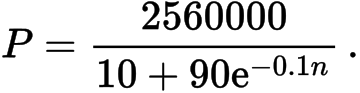
in the year 2007 ***A1 N3***

***[6 marks]***

**11b.** *[6 marks]*

At the end of 2000 there were  people in the city who used taxis.

After *n* years the number of people, *P*, in the city who used taxis is given by



(i) Find the value of *P* at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of *P* be double its value at the end of 2000? Justify your answer.

## Markscheme

(i)  ***(A1)***

 ***(A1)***

 ***A1 N3***

(ii) 

 ***A1***

not doubled ***A1 N0***

valid reason for **their** answer ***R1***

e.g. 

***[6 marks]***

**12a.** *[3 marks]*

Let  , for  .

Find the *x*-intercepts of the graph of *f* .

## Markscheme

intercepts when  ***M1***

(0.827, 0) (4.78, 0) (accept ,  ) ***A1A1 N3***

***[3 marks]***