Answer Key

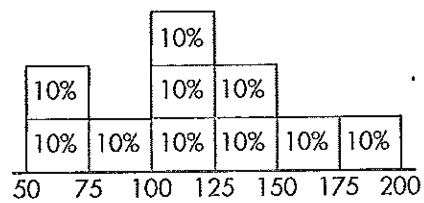
Section I

1. **E**	9. **D**	17. **B**	25. **C**	33. **E**
2. **E**	10. **D**	18. **A**	26. **C**	34. **A**
3. **A**	11. **C**	19. **B**	27. **D**	35. **C**
4. **D**	12. **D**	20. **E**	28. **D**	36. **E**
5. **B**	13. **A**	21. **E**	29. **C**	37. **C**
6. **A**	14. **B**	22. **E**	30. **A**	38. **C**
7. **C**	15. **D**	23. **B**	31. **C**	39. **A**
8. **D**	16. **A**	24. **E**	32. **D**	40. **B**

Answers Explained

Section I

1. **(E)**



The median price would have 50% of the prices below and 50% of the prices above; however, looking at areas, it is clear that 60% of the prices are below \$125,000. The mean (physically the center of gravity) appears to be less than \$125,000. Area considerations also show that 30% of the prices are between \$100,000 and \$125,000. Many values are too far from the mean for the standard deviation to be only \$10,000. If the distribution were closer to normal, the standard deviation would be around \$25,000; however, the distribution is more spread out than this, and with a SD perhaps between \$35,000 and \$40,000, estimating the variance at 1.5×10^9 is reasonable.

2. **(E)** Regression lines show association, not causation. Surveys suggest relationships, which experiments can help to show to be cause and effect.

3. **(A)** The critical z-score is 1.036. Thus $60 - 55 = 1.036\sigma$ and $\sigma = 4.83$.

4. **(D)** We are 90% confident that the population mean is within the interval calculated using the data from the sample.

5. **(B)** The first study was observational because the subjects were not chosen for treatment.

6. **(A)**

$$P(\text{def}) = P(\text{1st} \cap \text{def}) + P(\text{2nd} \cap \text{def})$$
$$= (.6)(.005) + (.4)(.010)$$
$$= .0030 + .0040 = .0070$$
$$P(\text{1st} \mid \text{def}) = \frac{.0030}{.0070} = .429$$

7. **(C)** At the dialysis center the more serious concern would be a Type II error, which is that the equipment is not performing correctly, yet the check does not pick this up; while at the towel manufacturing plant the more serious concern would be a Type I error, which is that the equipment is performing correctly, yet the check causes a production halt.

8. **(D)** There are two possible outcomes (heads and tails), with the probability of heads always .75 (independent of what happened on the previous toss), and we are interested in the number of heads in 10 tosses. Thus, this is a binomial model with $n = 10$ and $p = .75$. Repeating this over and over (in this case 50 times) simulates the resulting binomial distribution.

9. **(D)** In a binomial distribution with probability of success $p = \frac{5.1}{8.7} = .586$, the probability of at least 3 successes is

$$\binom{5}{3}(.586)^3(.414)^2 + \binom{5}{4}(.586)^4(.414)$$

$+ (.586)^5 = .658$, or using the TI-83, one finds that 1-binomcdf(5, .586, 2) $= .658$.

10. **(D)** If A and B are mutually exclusive, then $P(A \cap B) = 0$, and so $P(A \cup B) = .3 + .2 - 0 = .5$. If A and B are independent, then $P(A \cap B) = P(A)P(B)$, and so $P(A \cup B) = .3 + .2 - (.3)(.2) = .44$. If B is a subset of A, then $A \cup B = A$, and so $P(A \cup B) = P(A) = .3$.

11. **(C)** The standard deviation of the test statistic is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$. Since this is a two-sided test, the P-value will be twice the tail probability of the test statistic; however, the test statistic itself is not doubled.

12. **(D)** Dosage is the only explanatory variable, and it is being tested at three levels. Tumor reduction is the single response variable.

13. **(A)** The median corresponds to a cumulative proportion of 0.5.

14. **(B)** Standard deviation is a measure of variability. The less variability, the more homogeneity.

15. **(D)** Since the sample sizes are small, the samples must come from normally distributed populations. While the samples should be independent simple random samples, np and $n(1 - p)$ refer to conditions for tests involving sample proportions, not means.

16. **(A)** Adding the same constant to all values in a set will increase the mean by that constant, but will leave the standard deviation unchanged.

17. **(B)** With about 68% of the values within 1 standard deviation of the mean, the expected numbers for a normal distribution are as follows:

16	34	34	16

$$\text{Thus } \chi^2 = \sum \frac{(\text{obs} - exp)^2}{\text{exp}}$$

$$= \frac{(10-16)^2}{16} + \frac{(40-34)^2}{34} + \frac{(35-34)^2}{34} + \frac{(15-16)^2}{16}.$$

18. **(A)** This is a good example of voluntary response bias, which often overrepresents strong or negative opinions. The people who chose to respond were very possibly the parents of children facing drug problems or people who had had bad experiences with drugs being sold in their neighborhoods. There is very little chance that the 2500 respondents were representative of the population. Knowing more about his listeners or taking a sample of the sample would not have helped.

19. **(B)** The range (difference between largest and smallest values), the interquartile range ($Q_3 - Q_1$), and this difference between the 60th and 40th percentile scores all are measures of variability, or how spread out is the population or a subset of the population.

20. **(E)** Without independence we cannot determine var(X + Y) from the information given.

21. **(E)** The correlation coefficient r is not affected by changes in units, by which variable is called x or y, or by adding or multiplying all the values of a variable by the same constant.

22. **(E)** $1.96\left(\frac{.5}{\sqrt{n}}\right) \le .03$ gives $\sqrt{n} \ge 32.67$ and $n \ge 1067.1$.

23. **(B)** P(at least one Type I error) = 1 – P(no Type I errors) = $1 - (.95)^{10} = .40$.

24. **(E)** Note that all three sets have the same mean and the same range. The third set has most of its values concentrated right at the mean, while the second set has most of its values concentrated far from the mean.

25. **(C)** People coming out of a Wall Street office building are a very unrepresentative sample of the adult population, especially given the question under consideration. Using chance and obtaining a high response rate will not change the selection bias and make this into a well-designed survey. This is a convenience sample, not a voluntary response sample.

26. **(C)** Larger samples (so $\frac{\sigma}{\sqrt{n}}$ is smaller) and less confidence (so the critical z or t is smaller) both result in smaller intervals.

27. **(D)** The third scatterplot shows perfect negative association, so $r_3 = -1$. The first scatterplot shows strong, but not perfect, negative correlation, so $-1 < r_1 < 0$. The second scatterplot shows no correlation, so $r_2 = 0$.

28. **(D)** The probability of throwing heads is .5. By the law of large numbers, the more times you flip the coin, the more the relative frequency tends to become closer to this probability. With fewer tosses there is more chance for wide swings in the relative frequency.

29. **(C)** $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{27}{\sqrt{225}} = 1.8$. If the true mean parking duration is 51 minutes, the normal curve should be centered at 51. The critical value of 50 has a z-score of $\frac{50-51}{1.8}$.

30. **(A)** The overall lengths (between tips of whiskers) are the same, and so the ranges are the same. Just because the min and max are equidistant from the median, and Q_1 and Q_3 are equidistant from the median, does not imply that a distribution is symmetric or that the mean and median are equal. And even if a distribution is symmetric, this does not imply that it is roughly normal. Particular values, not distributions, may be outliers.

31. **(C)** Critical z-scores are $\frac{700-650}{45} = 1.11$ and $\frac{740-650}{45} = 2$ with right tail probabilities of .1335 and .0228, respectively. The percentage below 740 given that the scores are above 700 is $\frac{.1335-.0228}{.1335} = 82.9\%$.

32. **(D)** There is a different probability of Type II error for each possible correct value of the population parameter, and 1 minus this probability is the power of the test against the associated correct value.

33. **(E)** This is not a simple random sample because all possible sets of the required size do not have the same chance of being picked. For example, a set of principals all from just half the school districts has no chance of being picked to be the sample. This is not a cluster sample in that there is no reason to believe that each school district resembles the population as a whole, and furthermore, there was no random sample taken of the school districts. This is not systematic sampling as the districts were not put in some order with every nth district chosen. Stratified samples are often easier and less costly to obtain and also make comparative data available. In this case responses can be compared among various districts.

34. **(A)** A simple random sample can be any size and may or may not be representative of the population. It is a method of selection in which every possible sample of the desired size has an equal chance of being selected.

35. **(C)** The critical *z*-scores go from ±1.645 to ±2.576, resulting in an increase in the interval size: $\frac{2.576}{1.645} = 1.57$ or an increase of 57%.

36. **(E)** If the *P*-value is less than .10, it does not follow that it is less than .05. Decisions such as whether a test should be one- or two-sided are made before the data are gathered. If $\alpha = .01$, there is a 1% chance of rejecting the null hypothesis *if* the null hypothesis is true. There is one probability of a Type I error, the significance level, while there is a different probability of a Type II error associated with each possible correct alternative, so the sum does not equal 1.

37. **(C)** $\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ has a *t*-distribution with $df = n - 1$.

38. **(C)** A correlation of .6 explains $(.6)^2$ or 36% of the variation in *y*, while a correlation of .3 explains only $(.3)^2$ or 9% of the variation in *y*.

39. **(A)** Using a measurement from a sample, we are never able to say *exactly* what a population proportion is; rather we always say we have a certain *confidence* that the population proportion lies in a particular *interval*. In this case that interval is 43% ± 5% or between 38% and 48%.

40. **(B)** With Plan I the expected number of students with stock investments is only 2.4 out of 30. Plan II allows an estimate to be made using a full 30 investors.

SECTION II

1. A complete answer compares shape, center, and spread.

   Shape: The baseball players, (A), for which the cumulative frequency plot rises steeply at first, include more shorter players, and thus the distribution is skewed to the right (toward the greater heights). The football players, (C), for which the cumulative frequency plot rises slowly at first, and then steeply toward the end, include more taller players, and thus the distribution is skewed to the left (toward the lower heights). The basketball players, (B), for which the cumulative frequency plot rises slowly at each end, and steeply in the middle, have a more bell-shaped distribution of heights.

   Center: The medians correspond to relative frequencies of 0.5. Reading across from 0.5 and then down to the *x*-axis shows the median heights to be about 63.5 inches for baseball players, about 72.5 inches for basketball players, and about 79 inches for football players.

   Spread: The range of the football players is the smallest, 80 – 65 = 15 inches, then comes the range of the baseball players, 80 – 60 = 20 inches, and finally the range of the basketball players is the greatest, 85 – 60 = 25 inches.