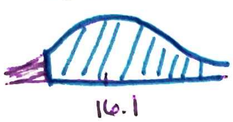
AP Statistics

Normal Distribution - Concept Check

Name: Answer Key

1. The packing machine at Kellogg's is set to fill a cardboard box of Frosted Flakes with a mean of 16.1 ounces of cereal. Suppose the amounts per box form a normal distribution with a standard deviation equal to 0.04 ounce. $N(16.1, 0.04)$

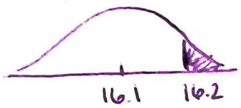
a. What percentage of the boxes will end up with at least 1 pound (16 ounces) of cereal?



$P(X>16) = P\left(Z > \frac{16-16.1}{0.04}\right) = 1 - .006 = \boxed{.9938}$

$P(Z>-2.5) =$ normalcdf(16, 1E99, 16.1, 0.04) =

b. What percentage of the boxes will have more than 16.2 ounces of cereal?



$P(X>16.2) = P\left(Z > \frac{16.2-16.1}{0.04}\right) = P(Z>2.5) = \boxed{.0062}$

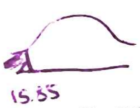
normalcdf(16.2, 1E99, 16.1, 0.04)

c. What percentage of the boxes will have between 15.95 and 16.15 ounces of cereal?

$P(15.95<X<16.15) =$ normalcdf(15.95, 16.15, 16.1, 0.04) =

$P\left(\frac{15.95-16.1}{0.04} < Z < \frac{16.15-16.1}{0.04}\right) = P(-3.75<Z<1.25) = \boxed{.8943}$

d. What percentage of the boxes will have less than 15.85 ounces of cereal?

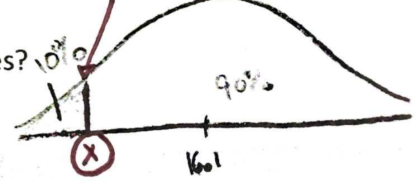


$P(X<15.85) = P\left(Z < \frac{15.85-16.1}{0.04}\right)$ normalcdf(-1E99, 15.85, 16.1, 0.04)

$P(Z<-6.25) = \boxed{.0000000002206}$

invNorm(.10, 0, 1) = -1.28

e. Ten percent of the boxes will contain **less** than what number of ounces?



$z = \frac{x-\mu}{\sigma}$ $\quad -1.28 = \frac{x-16.1}{.04}$ $\quad \boxed{X = 16.05}$

f. Eighty percent of the boxes will contain **more** than what number of ounces?

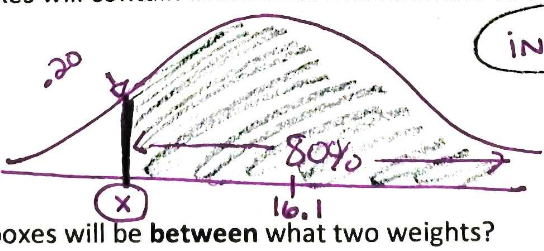
Z-score

invNorm(.2, 0, 1) = -.84

invNorm(.2, 16.1, 0.04) = $\boxed{16.0}$

$\downarrow$ μ σ

PLAN



Do: $\boxed{Z = \frac{X-\mu}{\sigma}}$

$-.84 = \frac{x-16.1}{0.04}$

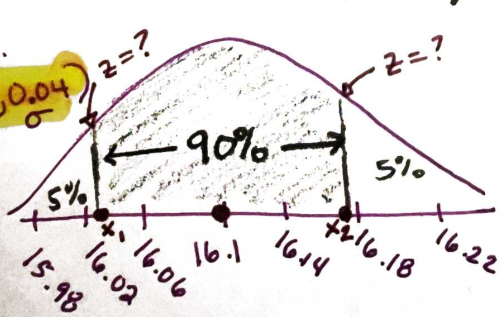
$\boxed{X = 16.07}$

g. The middle 90% of the boxes will be **between** what two weights?

STATE: FIND the box weights that the middle 90% of the boxes lie between.

$P(\;\; < X < \;\;) = .90$

PLAN: $N(16.1, 0.04)$ μ σ



Z-scores

invNorm(.05, 0, 1) = -1.64

invNorm(.95, 0, 1) = 1.64

$-1.64 = \frac{x-16.1}{0.04}$

$1.64 = \frac{x-16.1}{0.04}$

invNorm(.05, 16.1, 0.04) = $\boxed{16.03}$

invNorm(.95, 16.1, 0.04) = $\boxed{16.17}$

$\downarrow$ % $\quad \downarrow$ μ $\quad \downarrow$ σ

2. Assume that the distribution of scores on our Normal Distribution quiz is known to be normally distributed with a mean of 70 and a standard deviation of 5.3.

$N(70, 5.3)$

a. What is the probability that students scores between an 80 and an 85?

$P(80 < X < 85) = \boxed{.0273}$

b. What is the probability that a student fails? (below a 60%)

$P(X < 60) = \boxed{.0296}$

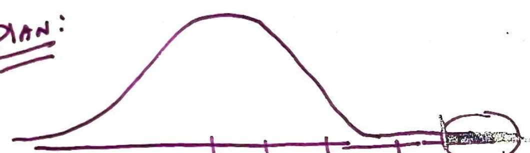
c. What is the probability that a student scores between a C+ (77) and a B+ (87)?

$P(77 < X < 87) = \boxed{.0926}$

STATE: $P(X > 90)$

d. What is the probability that a student scores an A? (above a 90) $N(70, 5.3)$

PLAN:



DO: $P\left(Z > \frac{90-70}{5.3}\right) = P(Z > 3.77)$

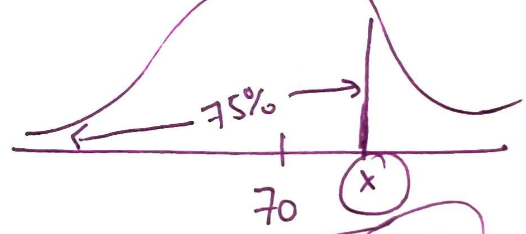
normalcdf(90, 1E99, 70, 5.3) = $\boxed{.0000804}$

μ, σ

e. What score represents the 3rd quartile?

70 75.3 80.6 85.9

$\boxed{73.57}$



f. What observation has 30% of the observations above it?

$\boxed{72.78}$

invNorm(.75, 70, 5.3)

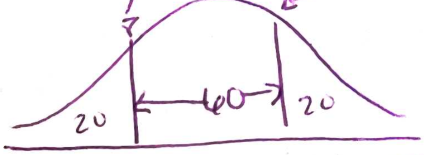
(.75, 0, 1)

Z-score

g. What scores have the middle 60% of the data between them?

invNorm(.2)

invNorm(.8)



$\boxed{65.54 \text{ \& } 74.46}$