Section 9.1 – Intro to Significance Tests – Guided Notes

Confidence intervals are used to ***estimate*** a parameter (*µ* or *p*) based on a sample statistic (𝑥̅ or 𝑝̂).

Statistical Inference

Significance Tests are used to ***evaluate a claim*** about a population parameter (*µ* or *p*) based on a sample statistic (𝑥̅ or 𝑝̂).

\*\*\*Used to reject or fail to reject explanations on the basis of their relative likelihood.

# Hypotheses:

The **Null Hypothesis** (*H*0) is a statement of “no difference.” This is the hypothesis we weigh evidence *against*. The **Alternative Hypothesis** (*H*a) is the claim we are trying to find evidence *for*.

\*indicated with greater than,

|  |  |  |  |
| --- | --- | --- | --- |
|  | Means |  | Proportions |
| **Null:** | H0: *µ* = 𝜇0 | or | H0: *p* = 𝑝0 |
| **Alternative**: |  |  |  |
| (one sided): | Ha: *µ* > 𝜇Ha: *µ* < 𝜇0 | oror | Ha: *p* > 𝑝0Ha: *p* < 𝑝0 |
|  |  |  |  |
| (two sided) | Ha: *µ* ≠ 𝜇0 | or | Ha: *p* ≠ 𝑝0 |

0 more than, or increased

\*indicated with less than, or decreased

\*indicated with a change or difference

**Check Your Understanding:** *For each of the following settings, (a) describe the parameter of interest, and (b) state appropriate hypotheses for a significance test.*

1. *According to the Web site sleepdeprivation.com, 85% of teens are getting less than eight hours of sleep a night. Janie wonders whether this result holds in her large high school. She asks an SRS of 100 students at the school how much sleep they get on a typical night. In all, 75 of the responders said less than 8 hours.*
	1. *p* =
	2. *H*0: *H*a:
2. *As part of its 2010 census marketing campaign, the U.S. Census Bureau advertised “10 questions, 10 minutes—that’s all it takes.” On the census form itself, we read, “The U.S. Census Bureau estimates that, for the average household, this form will take about 10 minutes to complete, including the time for reviewing the instructions and answers.” We suspect that the actual time it takes to complete the form may be longer than advertised.*
	1. *µ* =
	2. *H*0: *H*a:

**\*\*Common Exam Errors:** Don’t use statistics in the hypotheses. Statistics are found after you gather data, which you are doing to evaluate the claim you make in the hypotheses. Also, make sure you are using the symbols *p* or *µ* rather than 𝑝̂ or 𝑥̅, because we are trying to make inferences about the population parameter, not the sample statistic.

𝑥̅

𝑥̅

**Good:** *H*0: *µ* = 20 **Bad:** *H*0:

*H*a: *µ* > 20 *H*a:

= 20

> 20

Homework #1

1. Simon reads a newspaper report claiming that 12% of all adults in the United States are left-handed. He wonders if the proportion of lefties at his large community college is really 12%. Simon chooses an SRS of 100 students and records whether each student is right- or left-handed. Define the parameter and state the appropriate null hypothesis $H\_{0}$ and alternative hypothesis $H\_{a}$.
2. Birth weights In planning a study of the birth weights of babies whose mothers did not see a doctor before delivery, a researcher states the hypotheses as

$H\_{0}:\overbar{x}=1000$ grams

$H\_{a}:\overbar{x}<1000$ grams

Explain what’s wrong with the stated hypotheses. Then give correct hypotheses.

1. You are thinking about opening a restaurant and are searching for a good location. From research you have done, you know that the mean income of those living near the restaurant must be over $85,000 to support the type of upscale restaurant you wish to open. You decide to take a simple random sample of 50 people living near one potential

location. Based on the mean income of this sample, you will decide whether to open a restaurant there. State appropriate null and alternative hypotheses. Be sure to define your parameter.

1. Suppose you suspect a “chute” of playing cards is not fair. The chute supposedly contains 10 standard decks shuffled together. You are interested in knowing whether there are more hearts than usual. To test this, you deal 12 cards at random and calculate the proportion of hearts in your hand. Describe the parameter of interest in this setting and

write the appropriate null and alternative hypotheses for this situation.

1. Simon reads a newspaper report claiming that 12% of all adults in the United States are left-handed. He wonders if

12% of the students at his large public school are left-handed. Simon chooses an SRS of 100 students and records whether each student is right or left-handed. Describe the parameter of interest in this setting and write the

appropriate null and alternative hypotheses for this situation.

The ***P*-value** is the probability of an event happening assuming H0 is true. (We find this in the **DO** step)

*p* < 0.01 **strong evidence** to reject H0

> 99%

95% - 99%

90% - 95%

< 90%

How sig. tests are

related to confidence intervals

0.01 < *p* < 0.05 **enough evidence** to reject H0

0.05 < *p* < 0.10 **possible evidence** to reject H0

*P* > 0.10 **not enough evidence** to reject H0

**Example, p. 544 Healthy Bones** *Calcium is a vital nutrient for healthy bones and teeth. The National Institute of Health (NIH) recommends a calcium intake of 1300 mg per day for teenagers. The NIH is concerned that teenagers aren’t getting enough calcium. Is this true?*

*Researchers want to perform a test of*

*H*0: *µ* = 1300

*H*a: *µ* < 1300

*where µ is the true mean daily calcium intake in the population of teenagers. They ask a random sample of 20 teens to record their food and drink consumption for 1 day. The researchers then compute the calcium intake for each student. Data analysis reveals that* 𝑥̅ *=* 1198 *mg and sx* = 411 *mg. After checking that conditions were met, researchers performed a significance test and obtained a P-value of 0.1404.*

1. *Explain what it would mean for the null hypothesis to be true in this setting.*

In this setting, *H*0: *µ* = 1300 says that the mean daily calcium intake in the population of teenagers is 1300 mg. If *H*0 is true, then teenagers are getting enough calcium, on average.

1. *Interpret the P-value in context.*

Assuming that the mean daily calcium intake in the teen population is 1300 mg, there is a 0.1404 probability of getting a sample mean of 1198 mg or less just by chance in a random sample of 20 teens.

# \*\*Drawing conclusions based on the *P*-value:

**Example, p. 545 Healthy Bones**

The large *P*-value of 0.1404 gives weak evidence against *H*0. We therefore fail to reject *H*0. Researchers do

not have convincing evidence that teens are getting less than 1300 mg.

**Free Throws** *For Hypotheses: H0: p = 0.80 and Ha: p < 0.80 where p is the proportion of free throws a basketball player makes, a P-value of 0.0075 is calculated.*

The small *P*-value of 0.0075 gives strong evidence against *H*0. We therefore reject *H*0. We have convincing evidence that the player makes fewer than 80% of his free throws.

\*\*Sometimes, a significance level *α* is set. For instance, *α* = 0.05 or *α* = 0.01. This is the threshold at which we decide if we reject or fail to reject *H*0.

# Statistically Significant at level *α*

If the *P*-value is smaller than alpha, we say that the results of a study are **statistically significant at level *α*** In this case, we reject the null hypothesis *H*0, and conclude that there is convincing evidence in favor of the alternative hypothesis *H*a.

**Example, p. 546 Better batteries** *A company has developed a new deluxe AAA battery that is supposed to last longer than its regular AAA battery. However, these new batteries are more expensive to produce, so the company would like to be convinced that they really do last longer. Based on years of experience, the company knows that its regular AAA batteries last for 30 hours of continuous use, on average. The company selects an SRS of 15 new batteries and uses them continuously until they are completely drained. The sample mean lifetime is* 𝑥̅*= 33.9 hours. A significance test is performed using the hypotheses:*

*H*0: *µ* = 30 hours

*H*a: *µ* > 30 hours

*Where µ is the true mean lifetime of the new deluxe AAA batteries. The resulting P-value is 0.0729. What conclusion would you make for each of the following significance levels? Justify your answer.*

*(a) α* = 0.10

Because the *P*-value of 0.0729 is less than *α* = 0.10, we reject *H*0. We have convincing evidence that the company’s deluxe AAA batteries last longer than 30 hours, on average.

*(b) α* = 0.05

Because the *P*-value of 0.0729 is greater than *α* = 0.05, we fail to reject *H*0. We do not have convincing evidence that the company’s deluxe AAA batteries last longer than 30 hours, on average.

You do have the option of ***choosing*** a significance level if one is not stated. If you do this, make sure to do it

# before you begin the problem.

**\*\*\*EXAM TIP:** Conclusion to a significant test should include:

* 1. an explicit comparison of the *P*-value to a stated significance level,
	2. a decision about the null hypothesis: reject or fail to reject *H*0,
	3. a statement in the context of the problem about whether or not there is convincing evidence for *H*a.