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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

[redacted]

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 14]

The following table shows the average body weight, x, and the average weight of the brain, y, of seven species of mammal. Both measured in kilograms (kg).

Species	**Average body weight, x (kg)**	**Average weight of the brain, y (kg)**
Cat	3	0.026
Cow	465	0.423
Donkey	187	0.419
Giraffe	529	0.680
Goat	28	0.115
Jaguar	100	0.157
Sheep	56	0.175

(a) Find the range of the average body weights for these seven species of mammal. [2]

(b) For the data from these seven species

(i) calculate r, the Pearson's product–moment correlation coefficient;

(ii) describe the correlation between the average body weight and the average weight of the brain. [4]

(c) Write down the equation of the regression line y on x, in the form $y = mx + c$. [2]

The average body weight of grey wolves is 36 kg.

(d) Use your regression line to estimate the average weight of the brain of grey wolves. [2]

In fact, the average weight of the brain of grey wolves is 0.120 kg.

(e) Find the percentage error in your estimate in part (d). [2]

The average body weight of mice is 0.023 kg.

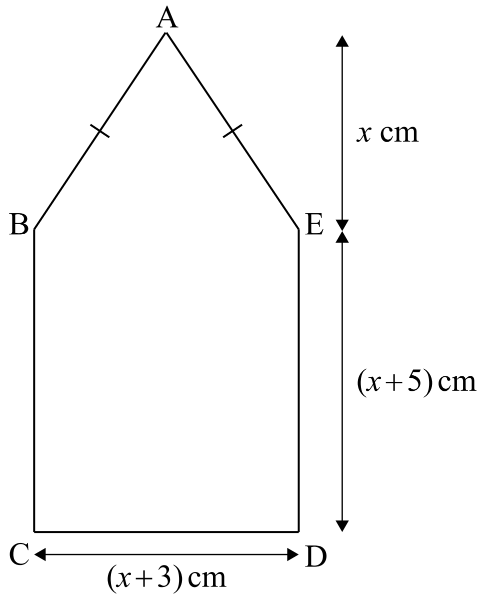
(f) State whether it is valid to use the regression line to estimate the average weight of the brain of mice. Give a reason for your answer. [2]

2. [Maximum mark: 16]

The base of an electric iron can be modelled as a pentagon ABCDE, where:

BCDE is a rectangle with sides of length $(x+3)\,\text{cm}$ and $(x+5)\,\text{cm}$;
ABE is an isosceles triangle, with $\text{AB} = \text{AE}$ and a height of x cm;
the area of ABCDE is $222\,\text{cm}^2$.

diagram not to scale



(a) (i) Write down an **equation** for the area of ABCDE using the above information.

(ii) Show that the equation in part (a)(i) simplifies to $3x^2 + 19x - 414 = 0$. [4]

(b) Find the length of CD. [2]

(c) Show that angle $\text{B}\hat{\text{A}}\text{E} = 67.4^\circ$, correct to one decimal place. [3]

Insulation tape is wrapped around the perimeter of the base of the iron, ABCDE.

(d) Find the length of the perimeter of ABCDE. [3]

F is the point on AB such that $\text{BF} = 8\,\text{cm}$. A heating element in the iron runs in a straight line, from C to F.

(e) Calculate the length of CF. [4]

3. [Maximum mark: 14]

Consider the function $f(x) = 0.3x^3 + \dfrac{10}{x} + 2^{-x}$.

(a) Calculate $f(1)$. [2]

(b) Sketch the graph of $y = f(x)$ for $-7 \le x \le 4$ and $-30 \le y \le 30$. [4]

(c) Write down the equation of the vertical asymptote. [2]

(d) Write down the coordinates of the x-intercept. [2]

(e) Write down the possible values of x for which $x < 0$ and $f'(x) > 0$. [2]

Consider a second function, $g(x) = 2x - 3$.

(f) Find the solution of $f(x) = g(x)$. [2]

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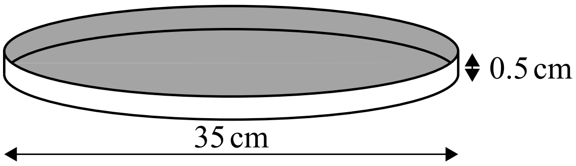
4. [Maximum mark: 15]

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.

diagram not to scale



(a) Calculate the volume of this pan. [3]

A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

(b) Find the radius of the sphere in cm, correct to one decimal place. [4]

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, $^\circ\text{C}$, can be modelled by

$$P(t) = a(2.06)^{-t} + 19,\ t \geq 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was $230\,^\circ\text{C}$.

(c) Find the value of a. [2]

(d) Find the temperature that the pizza will be 5 minutes after it is taken out of the oven. [2]

The pizza can be eaten once its temperature drops to $45\,^\circ\text{C}$.

(e) Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten. [3]

(f) In the context of this model, state what the value of 19 represents. [1]

5. [Maximum mark: 15]

The table below shows the distribution of test grades for 50 IB students at Greendale School.

Test grade	1	2	3	4	5	6	7
Frequency	1	3	7	13	11	10	5

(a) Calculate

(i) the mean test grade of the students;

(ii) the standard deviation. [3]

(b) Find the median test grade of the students. [1]

(c) Find the interquartile range. [2]

A student is chosen at random from these 50 students.

(d) Find the probability that this student scored a grade 5 or higher. [2]

A second student is chosen at random from these 50 students.

(e) Given that the first student chosen at random scored a grade 5 or higher, find the probability that both students scored a grade 6. [3]

The number of minutes that the 50 students spent preparing for the test was normally distributed with a mean of 105 minutes and a standard deviation of 20 minutes.

(f) (i) Calculate the probability that a student chosen at random spent at least 90 minutes preparing for the test.

(ii) Calculate the expected number of students that spent at least 90 minutes preparing for the test. [4]

6. [Maximum mark: 16]

Consider the function $g(x) = x^3 + kx^2 - 15x + 5$.

(a) Find $g'(x)$. [3]

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

(b) (i) Show that $k = 6$.

(ii) Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form $y = mx + c$. [5]

(c) Use your answer to part (a) and the value of k, to find the x-coordinates of the stationary points of the graph of $y = g(x)$. [3]

(d) (i) Find $g'(-1)$.

(ii) Hence justify that g is decreasing at $x = -1$. [3]

(e) Find the y-coordinate of the local minimum. [2]