

One-Sample z Tests - VS - One-Sample t Tests

1. The National Center for Health Statistics reports that the mean blood pressure for males 35-44 years of age is 128 and the standard deviation in this population is 15. The medical director of 3M looks at the medical records of a random sample of 72 executives in this age group and finds that the mean blood pressure in this sample is $\bar{x} = 126.07$. Is there significant evidence at the 5% level that 3M's executives have a different mean blood pressure from the general population? Assume that 3M has over 1000 male executives in this age group.

P: State what the *parameter* of interest is representing in this problem.

H: State *hypotheses* in words and symbols.

A: Verify the *assumptions/conditions*.

- Random.
- Normal.
- Independent.

N: Name the appropriate inference procedure.

T: Carry out the selected procedure. Find the *test statistic*.

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

O: Obtain the corresponding *P-value* based on the test statistic and H_a .

M: Make a decision to reject or fail to reject H_0 .

S: State your conclusion in the context of the problem.

2. When working properly, a soda machine will dispense soda with a mean of $\mu = 12$ ounces. A quality control engineer is concerned that the machine is over filling the cups. He draws a random sample of 8 cups and records the following amounts:

11.88 12.65 12.49 12.14 12.25 12.55 12.08 11.78

Is there evidence at the 5% level that the machine is over filling the cups?

P: State what the *parameter of interest* is representing in this problem.

H: State *hypotheses* in words and symbols.

A: Verify the *assumptions/conditions*.

- Random.
- Normal.
- Independent.

N: Name the appropriate inference procedure.

T: Carry out the selected procedure. Find the *test statistic*.

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

O: Obtain the corresponding *P-value* based on the test statistic and H_a .

M: Make a decision to reject or fail to reject H_0 .

S: State your conclusion in the context of the problem.

3. In a discussion of the education level of the American workforce, someone says, "The average young person can't even balance a checkbook." The NAEP survey says that a score of 275 or higher on its test reflects the skill needed to balance a checkbook. The NAEP random sample of 840 young Americans had a mean score of $\bar{x} = 272$, a bit below the checkbook-balancing level. If the standard deviation of scores of every young American on the test is $\sigma = 60$, is this sample result significant evidence at the 5% level that the mean score for *all* young Americans is less than 275?

4. A pharmaceutical manufacturer forms tablets by compressing a material that contains the active ingredient and various extras. The hardness of a sample from each lot of tablets produced is measured in order to control the compression process. The target values for the hardness are $\mu = 11.5$ and $\sigma = 0.2$. The hardness data for a sample of 20 tablets are

11.627	11.613	11.493	11.602	11.360	11.374	11.592	11.458	11.552	11.463
11.383	11.715	11.485	11.509	11.429	11.477	11.570	11.623	11.472	11.531

Is there significant evidence at the 5% level that the mean hardness of the tablets is different from the target value?

5. A hospital nurse is conducting a study about sleeping habits of four-year-olds. She wonders if they get more sleep than the recommended 8 hours per night. To test her claim, she collects a simple random sample of 12 four-year-olds and asks their parents how much sleep they got last night. Suppose the distribution of the amount of sleep for all four-year-olds is approximately Normal. The results are given below.

Child	Hrs of sleep
1	9.25
2	8.25
3	6.50
4	8.50
5	7.50
6	9.25
7	9.00
8	8.00
9	8.25
10	9.75
11	10.00
12	9.25

Conduct an appropriate test of significance (at the 10% level) to decide if the data support the nurse's claim that four-year-olds average more than 8 hours of sleep per night.

6. Bottles of Coca-Cola are supposed to contain 300 milliliters (ml) of cola. There is some variation from bottle to bottle because the filling machinery is not perfectly precise. The distribution of the contents is Normal with standard deviation $\sigma = 3$ ml. An inspector who suspects that the bottler is under filling the bottles measures the contents of six randomly selected bottles. The results are

299.4	297.7	301.0	298.9	300.2	297.0
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Is this convincing evidence at the 10% level that the mean contents of Coca-Cola bottles is less than the advertised 300 ml?

Paired t Tests

1. Researchers designed an experiment to study the effects of caffeine withdrawal. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During one 2-day period, subjects took capsules containing their normal caffeine intake. During another 2-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each 2-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in depression.

The table below contains data on the subjects' scores on the depression test. Higher scores show more symptoms of depression. For each subject, we calculated the difference in test scores following each of the two treatments (placebo - caffeine). We chose this order of subtraction to get mostly positive values.

Results of a caffeine-deprivation study			
Subject	Depression (caffeine)	Depression (placebo)	Difference (placebo - caffeine)
1	5	16	11
2	5	23	18
3	4	5	1
4	3	7	4
5	8	14	6
6	5	24	19
7	0	6	6
8	0	3	3
9	2	15	13
10	11	12	1
11	1	0	-1

Carry out a significance test at the 1% level to determine if being deprived of caffeine leads to an increase in depression.

2. The following table gives the ages of the husband and wife of a simple random sample of couples that have obtained a marriage licenses in Cumberland County, Pennsylvania in 1993.

Couple #	Husband	Wife		Couple #	Husband	Wife	
1	25	22		13	25	24	
2	25	32		14	23	22	
3	51	50		15	19	16	
4	25	25		16	71	73	
5	38	33		17	26	27	
6	30	27		18	31	36	
7	60	55		19	26	24	
8	54	57		20	62	60	
9	31	30		21	29	26	
10	54	47		22	31	23	
11	23	23		23	29	28	
12	34	39		24	35	36	

Conduct an appropriate test of significance at the 5% level to determine if there is evidence to support the claim that husbands tend to be older than their wives.