KEY

Choosing Types of Confidence Intervals

For each problem, identify the inference procedure that is needed to carry out the given interval. Then carry out that procedure.

1. A recent Gallup Poll conducted telephone interviews with a random sample of U.S. adults aged 18 and older. Data were obtained for 1000 people. Of these, 37% said that football is their favorite sport to watch on television. Construct and interpret an 80% confidence interval for p.

P: p = proportion of all U.S. adults aged 18 and older who say that football is their favorite sport to watch on television.

A: ✓Random – A random sample of U.S. adults aged 18 and older was obtained.

✓Normal – $n\hat{p} = 1000(.37) = 370 \geq 10$ ✓

$n(1-\hat{p}) = 1000(.63) = 630 \geq 10$ ✓ Approximately Normal

✓Independent – $n \leq \frac{1}{10}N$

$1000 \leq \frac{1}{10}N$

$10{,}000 \leq N$ ✓

There are more than 10,000 U.S. adults aged 18 and older.

N: One-proportion z interval

I: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$.37 \pm 1.282\sqrt{\frac{(.37)(.63)}{1000}}$ → $.37 \pm .0196$

$(.3504, .3896)$

C: We are 80% confident that the interval from 35.04% to 38.96% captures the proportion of all U.S. adults aged 18 and older who say that football is their favorite sport to watch on television.

2. A school counselor wants to know how smart the students in her school are. She gets funding from the principal to give an IQ test to an SRS of 60 of the over 1000 students in the school. The mean IQ score in the sample was 114.98 and the standard deviation was 14.80. Construct and interpret a 90% confidence interval for the mean IQ score of all students at the school.

P: μ = mean IQ score of all students at the school.

A: ✓Random – An SRS of 60 students was taken.

✓Normal – Approximately Normal by the CLT since $n \geq 30$ $(n = 60)$.

✓Independent – $n \leq \frac{1}{10}N$

$60 \leq \frac{1}{10}N$

$600 \leq N$ ✓

We are told that the population is more than 1000.

N: One-sample t interval

I: $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

$114.98 \pm 1.676\left(\frac{14.80}{\sqrt{60}}\right)$

114.98 ± 3.20

$(111.78, 118.18)$

$df = 60 - 1 = 59$ use $df = 50$ as a conservative estimate

C: We are 90% confident that the interval from 111.78 to 118.18 captures the mean IQ score of all students at the school.

3. A random digit dialing telephone survey of 880 drivers asked, "Recalling the last ten traffic lights you drove through, how many of them were red when you entered the intersections?" Of the 880 drivers, 171 admitted that at least one light had been red. Construct and interpret a 95% confidence interval for the population proportion.

P: p = proportion of all drivers who would admit that at least one light out of the last ten traffic lights was red when they entered the intersection.

A: ✓ Random – A random digit dialing machine was used

✓ Normal – $n\hat{p} = 880(.1943) = 170.984 \geq 10$ ✓
$n(1-\hat{p}) = 880(.8057) = 709.016 \geq 10$ ✓

Approximately Normal

✓ Independent – $n \leq \frac{1}{10}N$

$880 \leq \frac{1}{10}N$

$8{,}800 \leq N$

There are at least 8,800 drivers in the population of all drivers.

N: One-proportion z interval

I: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$.1943 \pm 1.96 \sqrt{\frac{(.1943)(.8057)}{880}}$

$.1943 \pm .0261$

$(.1682, .2204)$

C: We are 95% confident that the interval from 16.82% to 22.04% captures the proportion of all drivers who would admit that at least light was red when they entered the intersection.

4. Here are measurements (in millimeters) of a critical dimension on an SRS of 16 of the more than 200 auto engine crankshafts produced in one day:

224.120	224.001	224.017	223.982	223.989	223.961	223.960	224.089
223.987	223.976	223.902	223.980	224.098	224.057	223.913	223.999

$\bar{x} = 224.002$

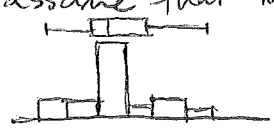
$s = .0618$

(a) Construct and interpret a 95% confidence interval for the mean measurement of all crankshafts produced in this day.

P: μ = mean measurement of all crankshafts produced in this day.

A: ✓Random – An SRS of 16 crankshafts was taken.

✓Normal – A graph of the sample data shows no outliers and no strong skewness, so it's safe to assume that the population might be Normal.



✓Independent – $n \le \frac{1}{10} N$

$16 \le \frac{1}{10} N$ There are more than 160 crankshafts produced in a day.

$160 \le N$

N: Since σ is unknown, we will use a one-sample t interval.

I: $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

$224.002 \pm 2.131 \left(\frac{.0618}{\sqrt{16}}\right) \rightarrow 224.002 \pm .0329$

$(223.9691, 224.0349)$

$df = 16 - 1 = 15$

C: We are 95% confident that the interval from 223.9691 mm to 224.0349 mm captures the mean measurement of all crankshafts produced in this day.

(b) The mean is supposed to be $\mu = 224$ mm but can drift away from this target during production. Does your interval from part (a) suggest that the mean has drifted away from this value? Explain.

No. Since 224 is within the interval, there's no evidence to suggest that the machine has drifted away from this value.

5. Suppose a student measuring the boiling temperature of a certain liquid observes the readings on 6 different samples of the liquid. She calculates the sample mean to be 101.82°C, and she knows that the distribution of all readings for this procedure is Normal with a standard deviation of 1.2 degrees Celsius. Construct and interpret the 99% confidence interval for the mean boiling temperature of all readings for this procedure.

P: μ = mean boiling temperature of <u>all</u> readings for this procedure.

A: Random - It is not stated that our samples were taken randomly.
It is <u>NOT</u> safe to continue with this interval.