KEY

Chapter 9 Review Sheet

1. The P-value of a test of significance is the probability:
   (a) the decision resulting from the test is correct.
   (b) 95% of the confidence intervals will contain the parameter of interest.
   (c) the null hypothesis is true.
   (d) the alternative hypothesis is true.
   (e) of obtaining a sample result as extreme or more extreme than the actual sample result assuming the null hypothesis is true.

2. If the P-value of a test of significance is greater than the level of significance, then which of the following conclusions are appropriate?
   I. The test is inconclusive.
   II. Accept the null hypothesis.
   III. The null hypothesis is true.
   (a) II only
   (b) III only
   (c) II and III
   (d) I only
   (e) None of the above.

3. Given $\alpha = 0.05$, which of the following is true?
   (a) P(Type II error) = 0.95
   (b) The power of the test is 0.95.
   (c) P(rejecting H_0 when H_0 is true) = 0.05
   (d) P(failing to reject H_0 when H_0 is false) = 0.05
   (e) The probability of a Type II error is independent of the value of α.

4. Durations of 74 storms in the Tampa Bay area are reported in a 2002 study. The mean is 75.7 min. The population standard deviation is 60.7 min. Assume the conditions for inference are met. A 95% confidence interval was constructed and found to be (61.87, 89.53). Suppose that we want to test the hypothesis H_0: $\mu = 85$ min versus H_a: $\mu \neq 85$ min using the data from above. Based on the confidence interval, what decision would you make about H_0? Be sure to include the significance level.

Fail to reject H_o at the 5% level since 85 is in the 95% confidence interval.

5. Suppose the mean weight of a rancher's cattle has been 400 lb with a standard deviation of 20 lb. A feed salesman has convinced him to try a new supplement on a random sample of 35 cattle. The mean weight of these cattle is 405 lb. Assume the rancher has more than 350 cattle.

a) Is there evidence at the $\alpha = 0.05$ significance level that cattle gain weight with the supplement?

P: μ = mean weight of the rancher's cattle with the supplement.

H: $H_0: \mu = 400$ The new supplement makes no difference in cattle weights.

$H_a: \mu > 400$ The cattle gain weight with the new supplement.

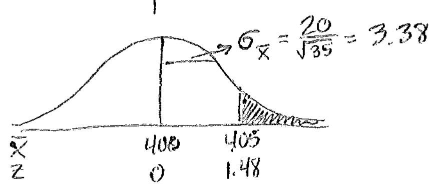
A: ✓Random - A random sample of 35 cattle is chosen.

✓Normal - Approximately Normal by the CLT since $n \geq 30$ $(n = 35)$.

✓Independent - $n \leq \frac{1}{10}N \rightarrow N \geq 350$ We are to assume the rancher has more than 350 cattle.

N: Since σ is known, we will use a one-sample z test

T: $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{405 - 400}{20/\sqrt{35}} \approx 1.48$



O: P-value = $P(z > 1.48) = .0694$

M: Because the P-value is not significant at the $\alpha = .05$ level, we fail to reject H_0.

S: There is not strong evidence that cattle gain weight with the supplement.

b) Describe a Type I error in the context of this problem.

We believe the cattle gain weight with the supplement, but the truth is that it makes no difference.

c) Describe a Type II error in the context of this problem.

We believe the supplement makes no difference but the truth is that cattle gain weight with the supplement.

d) Give two ways to reduce the probability of a Type I error.

Decrease the α-level

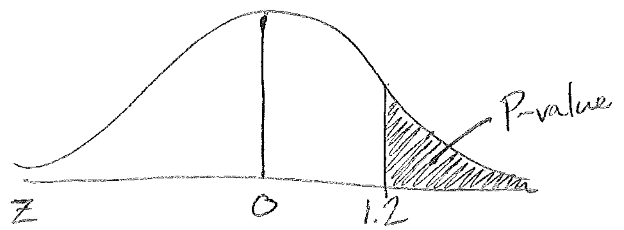
e) Give two ways to increase the Power of the test you performed.

Increase the α-level

Increase the sample size

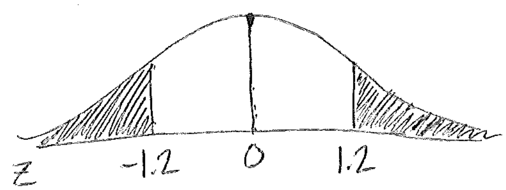
6. In a significance test of the hypotheses H_0: $\mu = 15$ versus H_a: $\mu > 15$, what is the P-value if the test statistic is $z = 1.2$? Make a sketch of the sampling distribution and shade the area under the curve that represents the P-value.

P-value = $P(z > 1.2) = .1151$



7. What is the P-value if the test statistic z is the same as in #6 but the alternative hypothesis is H_a: $\mu \neq 15$? Make a sketch of the sampling distribution and shade the area under the curve that represents the P-value.

P-value = $P(z < -1.2) + P(z > 1.2) = 2(.1151) = .2302$



8. Suppose the people of Lake Wobegon have always believed that the heights of their eighth grade girls is Normally distributed with a mean of 60 inches, and want to know if it is still 60 inches or if it is now greater than 60 inches. A sample of the heights of 23 randomly selected eighth grade girls gave a mean of 62.35. A one-sample t test with a one-sided alternative was done and the resulting P-value was 0.0498. Explain what 0.0498 is the probability of in the context of the problem.

There is a 4.98% chance that we got a sample mean of 62.35 or greater by chance alone, assuming the mean height of all eighth graders is still 60 inches.

9. A car dealership believes the percentage of its customers who are satisfied with their service is higher than the industry standard of 67%. Null and alternative hypotheses for testing this claim are given below. What error do these hypotheses contain?

H_0: $\hat{p} = .67$
H_a: $\hat{p} > .67$

The hypotheses are in terms of $\hat{p}$ instead of p.

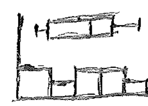
10. The effect of exercise on the amount of lactic acid in the blood was examined in the article "A Descriptive Analysis of Elite-Level Racquetball." Eight males were selected at random from those attending a large week-long training camp. Blood lactate levels were measured before and after playing three games of racquetball, as shown in the accompanying table. Is there significant evidence that blood lactate levels increase as a result of exercise?

Player	Before	After	After − Before
1	13	18	5
2	20	37	17
3	17	40	13
4	13	35	22
5	13	25	12
6	16	20	4
7	15	33	18
8	16	19	3

P: μ_d = mean difference (After − Before) in blood lactate levels among all males at this training camp.

H: $H_0: \mu_d = 0$ Blood lactate levels do not increase as a result of exercise

$H_a: \mu_d > 0$ Blood lactate levels increase as a result of exercise



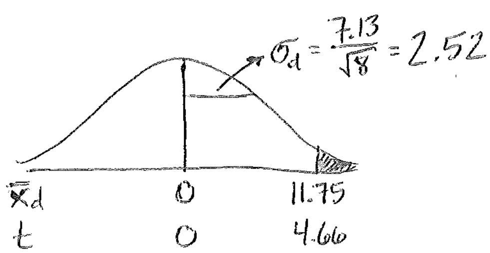
A: ✓Random – Eight males were selected at random.

✓Normal – A graph of sample data shows no outliers and no strong skewness.

✓Independent – $n \le \frac{1}{10}N \rightarrow N \ge 80$ We can assume that a large training camp has at least 80 males.

N: Paired t test

T: $t = \frac{\bar{x}_d - 0}{s_d/\sqrt{n}} = \frac{11.75}{7.13/\sqrt{8}} = 4.66$ $df = 8 - 1 = 7$



O: P-value = $P(t > 4.66)$ = between .001 and .0025

= .0012

M: Because the P-value is significant at the 5% level, we reject H_0.

S: There is strong evidence that blood lactate levels increase as a result of exercise.

11. Some boxes of a certain brand of breakfast cereal include a voucher for a free DVD rental inside the box. The company that makes the cereal claims that a voucher can be found in 20 percent of the boxes. However, based on their experiences eating this cereal at home, a group of students believes that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company's claim. The students found a total of 11 vouchers for free DVD rentals in the 65 boxes.

Suppose it is reasonable to assume that the 65 boxes purchased by the students are a random sample of all boxes of this cereal. Based on this sample, is there support for the students' belief at the 5% level that the proportion of boxes with vouchers is less than 0.2?

P: p = proportion of all boxes of this cereal with a free DVD rental voucher.

H: $H_0: p = 0.2$ The company's claim is correct

$H_a: p < 0.2$ The students' claim that the proportion is less than 0.2 is correct.

A: ✓ Random – Problem states that it's reasonable to assume the 65 boxes are a random sample.

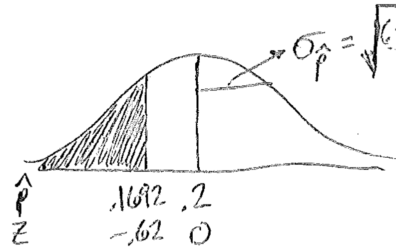
✓ Normal – $np_0 = 65(.2) = 13 \geq 10$ ✓ $n(1-p_0) = 65(.8) = 52 \geq 10$ ✓ Approximately Normal

✓ Independent – $n \leq \frac{1}{10}N \rightarrow N \geq 650$ It's safe to assume a company makes more than 650 boxes of a certain brand.

N: One-proportion z test

T: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{.1692 - .2}{\sqrt{\dfrac{(.2)(.8)}{65}}} = -.62$

$\sigma_{\hat{p}} = \sqrt{\dfrac{(.2)(.8)}{65}} = .0496$



O: P-value $= P(z < -.62) = .2676$

M: Because the P-value is not significant at the 5% level, we fail to reject H_0.

S: There is not strong evidence that the proportion of boxes with free DVD vouchers is less than 0.2.

Chapter 9 Learning Targets:

__ I can create hypotheses for a significance test.

__ I can calculate a P-value and explain its meaning.

__ I can use a P-value and significance level (α-level) to make a decision in a significance test.

__ I can explain what "statistically significant" means and determine whether a result is "statistically significant."

__ I can carry out a one-proportion z test.

__ I can carry out a one-sample z test.

__ I can carry out a one-sample t test.

__ I can carry out a paired t test.

__ I can decide which z-scores or t-scores will reject the null hypothesis based on a given α-level.

__ I can use a confidence interval instead of a P-value to make a decision for a two-sided significance test.

__ I can explain what Type I and Type II errors are and state each error in context.

__ I can find the probability of a Type I error.

__ I can describe what Power is.

__ I can explain the relationship between Power and the probability of a Type II error.

__ I can give ways to decrease the probability of errors and increase the Power of a significance test.