Chapter 7 Sampling Distributions Practice
AP Statistics

Name: Key

Things you should know and know when it's used:

- 10% condition → $n \leq \frac{1}{10}N$ n = sample size, N = population size

  *Used to see if the population is independent so we can use standard deviation. $(\sigma_{\bar{x}})(\sigma_{\hat{p}})$

- Large counts condition

  $np \geq 10$ and $n(1-p) \geq 10$ p = population proportion

  *Used to check for Normality with sample proportions

- Central Limit Theorem

  $n \geq 30$ *Used to check for Normal approximation with sample means if the population isn't Normal.

I. Multiple Choice. Select the answer that best completes the statement or answers the question.

1. The variability of a statistic is described by

(a) the spread of its sampling distribution.
(b) the amount of bias present.
(c) the vagueness in the wording of the question used to collect the sample data.
(d) probability calculations.
(e) the stability of the population it describes.

A

2. According to a recent poll, 27% of Americans prefer to read their news in a physical newspaper instead of online. Let's assume this is the parameter value for the population. If you take a simple random sample of 25 Americans and let $\hat{p}$ = the proportion in the sample who prefer a newspaper, is the shape of the sampling distribution of $\hat{p}$ approximately Normal? Check Large Counts!

$25(.27) = 6.75 \geq 10$ NO! $25(.73) = 18.25$

(a) No, because $p < 0.50$.
(b) No, because $np = 6.75$.
(c) Yes, because we can reasonably assume there are more than 250 individuals in the population.
(d) Yes, because we took a simple random sample.
(e) Yes, because $n(1-p) = 18.25$.

B

3. The time it takes students to complete a statistics quiz has a mean of 20.5 minutes and a standard deviation of 15.4 minutes. What is the probability that a random sample of 40 students will have a mean completion time greater than 25 minutes?

$\mu = 20.5$ $\sigma = 15.4$ $n = 40$ $\mu_{\bar{x}} = 20.5$

$\sigma_{\bar{x}} = \frac{15.4}{\sqrt{40}} \approx 2.43$

$P(\bar{x} > 25) = P\left(z > \frac{25-20.5}{2.43}\right)$
$= P(z > 1.85)$
$= 1 - 0.9678 = 0.0322$

(a) 0.9678
(b) 0.0322
(c) 0.0344
(d) 0.3851
(e) 0.6149

B

4. A fair coin (one for which both the probability of heads and the probability of tails are 0.5) is tossed 60 times. The probability that more than 1/3 of the tosses are heads is closest to

$p = 0.5 \quad n = 60$

(a) 0.9951
(b) 0.33
(c) 0.109
(d) 0.09
(e) 0.0049

A

$P(\hat{p} > 1/3) = P\left(z > \frac{1/3 - 0.5}{0.0645}\right)$
$= P(z > -2.58)$
$= 1 - 0.0049 = \boxed{0.9951}$

$\mu_{\hat{p}} = 0.5$

$\sigma_{\hat{p}} = \sqrt{\frac{(0.5)(0.5)}{60}} \approx 0.0645$

5. The incomes in a certain large population of college teachers have a normal distribution with mean \$60,000 and standard deviation \$5000. Four teachers are selected at random from this population to serve on a salary review committee. What is the probability that their average salary exceeds \$65,000?

$N(60{,}000, 5000)$ $\quad \bar{x}$

$n = 4$

$\mu_{\bar{x}} = 60{,}000$

$\sigma_{\bar{x}} = \frac{5000}{\sqrt{4}} = 2500$

$P(\bar{x} > 65{,}000)$
$= P\left(z > \frac{65{,}000 - 60{,}000}{2500}\right)$
$= P(z > 2) = 1 - 0.9772$
$= 0.0228$

(a) 0.0228
(b) 0.1587
(c) 0.8413
(d) 0.9772
(e) essentially 0

A

6. A random sample of size 25 is to be taken from a population that is Normally distributed with mean 60 and standard deviation 10. The mean $\bar{x}$ of the observations in our sample is to be computed. The sampling distribution of $\bar{x}$

$n = 25 \quad N(60, 10)$

$\sigma_{\bar{x}} = \frac{10}{\sqrt{25}} = 2$

(a) has an unknown shape with mean 60 and standard deviation 10.
(b) has an unknown shape with mean 60 and standard deviation 2.
(c) is Normal with mean 60 and standard deviation 10.
(d) is Normal with mean 60 and standard deviation 2.
(e) is approximately Normal with mean 60 and standard deviation 2.

D

Normal since population is Normal.

7. The scores of individual students on a college entrance examination have a left-skewed distribution with mean 18.6 and standard deviation 6.0. At Millard North High School, 36 seniors take the test. The sampling distribution of mean scores for random samples of 36 students is

$\mu = 18.6 \quad \sigma = 6.0 \quad n = 36$

(a) skewed right.
(b) symmetric and mound-shaped, but non-Normal.
(c) approximately Normal.
(d) neither Normal nor non-normal. It depends on the particular 36 students selected.
(e) exactly Normal.

C

By CLT, $36 \geq 30$, so approx. Normal

8. The distribution of prices for home sales in Minnesota is skewed to the right with a mean of \$290,000 and a standard deviation of \$145,000. Suppose you take a simple random sample of 100 home sales from this (very large) population. What is the probability that the mean of the sample is above \$325,000?

$\mu = 290{,}000 \quad \sigma = 145{,}000 \quad n = 100$

$\mu_{\bar{x}} = 290{,}000$

$\sigma_{\bar{x}} = \frac{145{,}000}{\sqrt{100}} = 14{,}500$

(a) 0.0015
(b) 0.0027
(c) 0.0079 closest
(d) 0.4046
(e) 0.4921

C

$P(\bar{x} > 325{,}000) = P\left(z > \frac{325{,}000 - 290{,}000}{14{,}500}\right)$
$= P(z > 2.41)$
$= 1 - 0.9920 = 0.0080$

Multiple Choice Answers: 1. A, 2. B, 3. B, 4. A, 5. A, 6. D, 7. C, 8. C

Free Response. *Answer each of the following completely. Show all of your steps (including verifying that you can use the normal approximation and standard deviation formula). Remember to use* ***context!!!!***

9. The Wechsler Adult Intelligence Scale (WAIS) is a common "IQ test" for adults. The distribution of WAIS scores for persons over 16 years of age is approximately Normal with mean 100 and standard deviation 15.

$N(100, 15)$

(a) What is the probability that a randomly chosen individual has a WAIS score of 105 or higher? Show your work.

$$P(x > 105) = P\left(z > \frac{105-100}{15}\right) = P(z > 0.33)$$
$$= 1 - 0.6293$$
$$= \boxed{0.3707}$$

(b) Find the mean and standard deviation for the sampling distribution of the average WAIS score $\bar{x}$ for an SRS of 60 people. $n = 60$

$\bar{x}$

$$\boxed{\mu_{\bar{x}} = 100}$$

$$\boxed{\sigma_{\bar{x}} = \frac{15}{\sqrt{60}} \approx 1.936}$$

Check 10% condition
$60 \le \frac{1}{10}$ (WAIS scores for persons over 16)
$600 \le$
Safe to assume.

(c) What is the probability that the average WAIS score of an SRS of 60 people is 105 or higher?

$$P(\bar{x} > 105) = P\left(z > \frac{105-100}{1.936}\right) = P(z > 2.58)$$
$$= 1 - 0.9951$$
$$= \boxed{0.0049}$$

(d) Would you answers to any of parts (a), (b), or (c) be affected if the distribution of WAIS scores in the adult population were distinctly non-Normal? Explain.

(a) would change because we calculated the probability using Normal calculations. (b) & (c) would not change because by the Central Limit Theorem ($n \ge 30$) allows us to use Normal approximation.

10. According to a market research firm, 52% of all residential telephone numbers in Los Angeles are unlisted. A telephone sales firm uses random digit dialing equipment that dials residential numbers at random, whether or not they are listed in the telephone directory. The firm calls 500 members in L.A.. What is the probability that at least half the numbers dialed are unlisted?

$p = 0.52$ $n = 500$

0.5

Check Large Counts Condition
$500(0.52) = 260 \ge 10$ ✓ $500(0.48) = 240 \ge 10$ ✓
Can use Normal approximation.

$\mu_{\hat{p}} = 0.52$

$\sigma_{\hat{p}} = \sqrt{\frac{(0.52)(0.48)}{500}} = 0.0223$

$$P(\hat{p} > 0.5) = P\left(z > \frac{0.5-0.52}{0.0223}\right) = P(z > -0.90)$$
$$= 1 - 0.1841 = \boxed{0.8159}$$

Check 10% condition:
$500 \le \frac{1}{10}$ (Los Angeles)
Safe to assume

11. The Gallup Poll once asked a random sample of 1470 adults, "Do you happen to jog?" Suppose that in fact 25% of all adults jog.

$p = 0.25$ $n = 1470$

(a) Find the mean and standard deviation of the proportion $\hat{p}$ of the sample who jog. (Assume the sample is an SRS.)

$\mu_{\hat{p}} = 0.25$ $\sigma_{\hat{p}} = \sqrt{\frac{(0.25)(0.75)}{1470}} \approx 0.0113$

(b) Explain why you can use the formula for the standard deviation of $\hat{p}$ in this setting.

Because $1470 \leq \frac{1}{10}$(all adults) is satisfied.
(10% condition)

(c) Check that you can use the normal approximation for the distribution of $\hat{p}$.

Large Counts condition Safe to use.

$1470(0.25) = 367.5 \geq 10$ ✓ $1470(0.75) = 1102.5 \geq 10$ ✓

(d) Find the probability that between 22% and 28% of the sample jog.

$P(0.22 \leq \hat{p} \leq 0.28) = P\left(\frac{0.22-0.25}{0.0113} \leq z \leq \frac{0.28-0.25}{0.0113}\right)$

$= P(-2.65 \leq z \leq 2.65) = 0.9960 - 0.0040 = \boxed{0.9920}$

(e) What sample size would be required to reduce the standard deviation of the sample proportion to one-third the value you found in (a)?

$n = ?$

$\sigma_{\hat{p}} = \frac{1}{3}\sqrt{\frac{(0.25)(0.75)}{1470}} = \sqrt{\frac{(0.25)(0.75)}{9 \cdot 1470}}$

$\downarrow$

$\boxed{13,230}$

12. The gypsy moth is a serious threat to oak and aspen trees. A state agriculture department places traps throughout the state to detect the moths. Each month, an SRS of 50 traps is inspected, the number of moths in each trap is recorded, and the mean number of moths is calculated. Based on years of data, the distribution of moth counts is discrete and strongly skewed, with a mean of 0.5 and a standard deviation of 0.7. $n = 50$

$\mu = 0.5$ $\sigma = 0.7$

(a) Explain why it is reasonable to use a Normal distribution to approximate the sampling distribution of $\bar{x}$ for SRSs of size 50.

Because $50 \geq 30$, by Central Limit Theorem the sampling distribution of $\bar{x}$ will be approximately Normal.

(b) Estimate the probability that the mean number of moths in a sample of size 50 is greater than or equal to 0.6.

$\mu_{\bar{x}} = 0.5$ $\sigma_{\bar{x}} = \frac{0.7}{\sqrt{50}} \approx 0.0990$

$P(\bar{x} \geq 0.6) = P\left(z \geq \frac{0.6-0.5}{0.0990}\right) = P(z \geq 1.01)$

$= 1 - 0.8438$

$= \boxed{0.1562}$