Chapter 5 Probability Review **AP Statistics** **Name:** Key

- Define Probability. The proportion of times an outcome would occur in a long series of repetitions.
- Rules of Probability:

1. Probability P(**A**) of any event **A** satisfies $0 \le P(\mathbf{A}) \le 1$. *in the long run
2. If **S** is the sample space in a probability model, then P(**S**) = 1 (Probabilities of all outcomes must add up to 1.)
3. The **complement** of any event **A** is the event **A** does not occur (A^C). $P(A^C) = 1 - P(A)$.
4. Events **A** & **B** are **mutually exclusive** (also known as disjoint) if they have no outcomes in common.
   Addition Rule: P(**A** or **B**) = P(**A**) + P(**B**)
5. **General Addition Rule:** For any 2 events A & B, P(A or B) = P(A) + P(B) – P(A and B)
6. **Conditional Probability:** For any events **A** & **B**, independent or dependent $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$
7. For **independent** events A & B: P(A | B) = P(A)

I. Multiple Choice Practice.

1. Suppose $P(X) = 0.25$ and $P(Y) = 0.40$. If $P(X \mid Y) = 0.20$, what is $P(Y \mid X)$?

(a) 0.10
(b) 0.125
(c) 0.32
(d) 0.45
(e) 0.50

C

$$P(X\mid Y) = \frac{P(X \cap Y)}{P(Y)}$$
$$0.20 = \frac{P(X\cap Y)}{0.40}$$
$$0.08 = P(X \cap Y)$$

$$P(Y\mid X) = \frac{P(X \cap Y)}{P(X)}$$
$$P(Y \mid X) = \frac{0.08}{0.25}$$
$$P(Y \mid X) = 0.32$$

2. The Air Force receives 40 percent of its parachutes from company C_1 and the rest from company C_2. The probability that a parachute will fail to open is 0.0025 or 0.002, depending on whether it is from company C_1 or C_2, respectively. If a randomly chosen parachute fails to open, what is the probability that it is from company C_1?

(a) 0.0010
(b) 0.0022
(c) 0.4025
(d) 0.4545
(e) 0.5455

D

.4 C_1 — .0025 F — .4×.0025 = .001
.4 C_1 — .9975 O — .4×.9975 = .399
.6 C_2 — .002 F — .0012
.6 C_2 — .998 O — .5988

$$P(C_1 \mid F) = \frac{P(C_1 \cap F)}{P(F)} = \frac{.001}{.001 + .0012} \approx 0.455$$

3. Given two events, E and F, such that $P(E) = 0.340$, $P(F) = 0.450$, and $P(E \cup F) = 0.637$, then the two events are

(a) Independent and mutually exclusive – can't be both
(b) Independent, but not mutually exclusive
(c) ~~Mutually exclusive~~, but not independent
(d) Neither independent nor mutually exclusive
(e) There is not enough information to answer this question.

B

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$.637 = .340 + .450 - P(E \cap F)$$
$$.153 = P(E \cap F)$$ ← have an intersection, not ME

Does $P(E) \cdot P(F) = P(E \cap F)$?
$(.34)(.45) = .153$ ✓ Independent!

4. Suppose you toss a fair coin ten times and it comes up heads every time. Which of the following is a true statement?

(a) By the Law of Large Numbers, the next toss is more likely to be tails than another heads.
(b) By the properties of conditional probability, the next toss is more likely to be heads given that ten tosses in a row have been heads.
(c) Coins actually do have memories, and thus what comes up on the next toss is influenced by the past tosses.
(d) The Law of Large Numbers tells how many tosses will be necessary before the percentages of heads and tails are again in balance.
(e) None of the above are true statements.

E

5. If P(A) = 0.25 and P(B) = 0.34, what is P($A \cup B$) if A and B are independent?

P(A) · P(B) = P(A and B)

(a) 0.085
(b) 0.505
(c) 0.590
(d) 0.675
(e) Insufficient information

B

0.25 · 0.34 = 0.085 = P(A and B)

P(AUB) = 0.25 + 0.34 − 0.085 = 0.505 (P(A), P(B), P(A and B))

6. China has 1.2 billion people. Marketers want to know which international brands they have heard of. A large study showed that 62% of all Chinese adults have heard of Coca-Cola. You want to simulate choosing a Chinese at random and asking if he or she has heard of Coca-Cola. One correct way to assign random digits to simulate the answer is:

(a) One digit simulates one person's answer; odd means "Yes" and even means "No."
(b) One digit simulates one person's answer; 0 to 6 mean "Yes" and 7 to 9 mean "No."
(c) One digit simulates the result; 0 to 9 tells how many in the sample said "Yes."
(d) Two digits simulate one person's answer; 00 to 61 mean "Yes" and 62 to 99 mean "No."
(e) Two digits simulate one person's answer; 00 to 62 mean "Yes" and 63 to 99 mean "No."

D

7. For the following probability model, what would the P(Yellow) have to be for the model to be legitimate?

Color (X)	Red	Blue	Green	Yellow	Orange
P(*X*)	.24	.21	.20	?	.17

A (a) 0.18 (b) 0.20 (c) 0.82 (d) 0.25 (e) 0.17

8. Using the probability model from question 7, what is the probability of **not** getting a red?

C (a) 0.24 (b) 0.20 (c) 0.76 (d) 0.18 (e) 1.00

9. A deck of playing cards has 52 cards, of which 12 are face cards. If you shuffle the deck well and turn over the top 3 cards, one after the other, what's the probability that all 3 are face cards?

C (a) 0.001 (b) 0.005 (c) 0.010 (d) 0.012 (e) 0.02

10. Choose an American adult at random. The probability that you choose a woman is 0.52. The probability that the person you choose has never married is 0.25. The probability that you choose a woman who has never married is 0.11. The probability that the person you choose is either a woman or has never been married (or both) is therefore about

0.52 + 0.25 − 0.11

B (a) 0.77 (b) 0.66 (c) 0.44 (d) 0.38 (e) 0.13

11. Dr. Stats plans to toss a fair coin 10,000 times in the hope that it will lead him to a deeper understanding of the laws of probability. Which of the following statements is true?

(a) It is unlikely that Dr. Stats will get more than 5000 heads.
(b) Whenever Dr. Stats gets a string of 15 tails in a row, it becomes more likely that the net toss will be a head.
(c) The fraction of tosses resulting in heads should be exactly ½.
(d) The chance that the 100th toss will be a head depends somewhat on the results of the first 99 tosses.
(e) It is likely that Dr. Stats will get about 50% heads.

E

12. computer voice recognition software is getting better. Some companies claim that their software correctly recognizes 98% of all words spoken by a trained user. To simulate recognizing a single word when the probability of being correct is 0.98, let two digits simulate one word; 00 to 97 mean "correct." The program recognizes words (or not) independently. To simulate the program's performance on 10 words, use these random digits:

B

60970 70024 17868 29843 61790 90656 87964
C C C C C C C C X C

The number of words recognized correctly out of the 10 is:

(a) 10 (b) 9 (c) 8 (d) 7 (e) 6

Multiple Choice Answers: 1. C, 2. D, 3. B, 4. E, 5. B, 6. D, 7. A, 8. C, 9. C, 10. B, 11. E, 12. B

Free Response.

13. Police report that 78% of drivers stopped on suspicion of drunk driving are given a breath test, 36% a blood test, and 22% both tests.

A = Breath   B = Blood

(a) Using the General Addition Rule, find the probability that a randomly selected DWI suspect is given a blood test or a breath test.

$P(A) + P(B) - P(A \text{ and } B)$

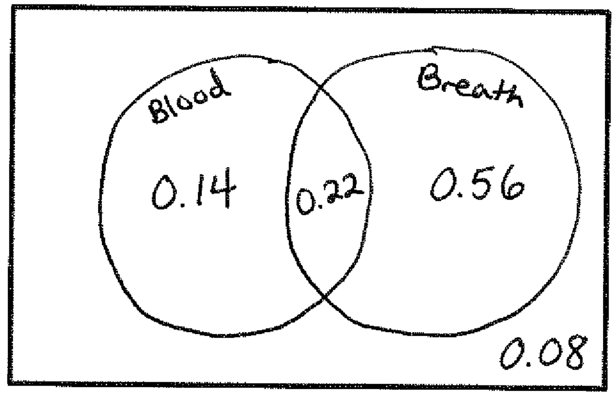
$P(A \text{ or } B) = 0.78 + 0.36 - 0.22 = \boxed{0.92}$

(b) Represent this situation in a two-way table.

Breath

Blood		Yes	No	Total
	Yes	0.22	0.14	0.36
	No	0.56	0.08	0.64
	Total	0.78	0.22	1.00

(c) Represent this situation in a Venn Diagram



(d) Find the probability that a randomly selected DWI suspect is given:

1. P(either test) one or both — $P(A \text{ or } B) = \boxed{0.92}$
2. P(only a blood test) — $P(A^c \text{ and } B) = \boxed{0.14}$
3. P(only a breath test) — $P(A \text{ and } B^c) = \boxed{0.56}$
4. P(neither test) — $P(A^c \text{ and } B^c) = \boxed{0.08}$

(e) Are the tests independent? Use probability rules to support your answer.

Check $P(A) \cdot P(B) = P(A \text{ and } B)$

$0.78 \cdot 0.36 \stackrel{?}{=} 0.22$

$0.2808 \neq 0.22$

No, they are not independent.

14. Suppose you have a standard deck of 52 cards. Find the probability of drawing a certain card given the following events: A = {draw a diamond}, B = {draw a black card}, C = {draw a 4}

(a) P(A): $\frac{13}{52} = \boxed{\frac{1}{4} = 0.25}$

(b) P(B): $\frac{26}{52} = \boxed{\frac{1}{2} = 0.5}$

(c) P(C): $\frac{4}{52} = \boxed{\frac{1}{13} \approx 0.077}$

(d) P(A^C): $\frac{39}{52} = \boxed{\frac{3}{4} = 0.75}$

(e) P(A ∪ B) no black diamonds: $\frac{1}{4} + \frac{1}{2} - 0 = \boxed{\frac{3}{4} = 0.75}$

(f) P(A^C ∩ B) not a diamond: $\frac{26}{52} = \boxed{\frac{1}{2} = 0.5}$

(g) P(B^C): $1 - P(B) = 1 - \frac{1}{2} = \boxed{\frac{1}{2} = 0.5}$

(h) P(B^C ∪ C) red: $\frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \boxed{\frac{7}{13} \approx 0.538}$

(i) P(C | B): $\frac{2}{26} = \boxed{\frac{1}{13} \approx 0.077}$

(j) P(B | C): $\frac{2}{4} = \boxed{\frac{1}{2} = 0.5}$

15. The probability that Travis makes a free throw is 0.6. Each shot is independent of the others.

(a) What is the probability that Travis makes three free throws in a row?

$0.6 \cdot 0.6 \cdot 0.6 = \boxed{0.216}$

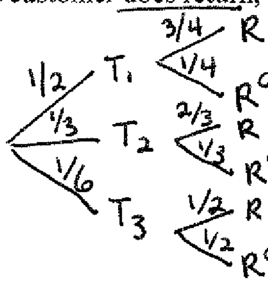
(b) What is the probability that Travis misses one free throw?

$1 - 0.6 = \boxed{0.4}$

(c) Travis' coach has asked him to shoot 10 free throws before the end of practice. What is the probability that Travis will make all 10 free throws? What is the probability that he will miss all 10 free throws?

$P(\text{all}) = (0.6)^{10} \approx \boxed{0.0060}$ $\qquad P(\text{miss all}) = (0.4)^{10} \approx \boxed{0.0001}$

16. A travel agent books passages on three different tours, with half her customers choosing tour one (T_1), one-third choosing tour two (T_2), and the rest choosing tour three (T_3). The agent noted that three-quarters of those who take tour one return to book passage again, two-thirds of those who take tour two return, and one-half of those who take tour three return. If a customer does return, what is the probability that the person first went on tour two? (Use a tree diagram.)



$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

$\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$

$\frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$

$$P(T_2 \mid R) = \frac{P(T_2 \cap R)}{P(R)}$$

$$= \frac{2/9}{\frac{3}{8} + \frac{2}{9} + \frac{1}{12}} \approx \boxed{0.3265}$$

17. All human blood can be "ABO-typed" as one of O, A, B, or AB, but the distribution of the types varies a bit among groups of people. Here is the distribution of blood types for a randomly chosen person in the United States:

Blood Type:	O	A	B	AB
U.S. probability:	0.45	0.40	0.11	?

(a) What is the probability of type AB blood in the United States? Why?

0.04, Because $0.45 + 0.4 + 0.11 + 0.04 = 1.00$.

(b) An individual with type B blood can safely receive transfusions only from persons with type B or type O blood. What is the probability that the husband of a woman with type B blood is an acceptable blood donor for her?

$0.11 + 0.45 = \boxed{0.56}$

(c) What is the probability that in a randomly chosen couple the wife has type B blood and the husband has type A?

independent

$(0.11)(0.40) = \boxed{0.044}$

18. Based on previous records, 17% of the vehicles passing through a tollbooth have out-of-state plates. A bored tollbooth worker decides to pass the time by counting how many vehicles pass through until he sees two with out-of-state plates.

(a) Describe the design of a simulation to estimate the average number of vehicles it takes to find two with out-of-state plates. Explain clearly how you will use the partial table of random digits below to carry out your simulation.

Let 00-16 represent out-of-state plates and 17-99 be in-state plate. Reading two digits at a time, we will look at cars until we get two ~~one~~ out of state plates. We will record the number of cars it takes to get ~~two~~ to ~~one~~ out of state plates.

(b) Perform three repetitions of the simulation you describe in part(a). Copy the random digits below onto your paper. Then mark on or directly above the table to show your results. ✓ = out x = in

32 14 13

41050	92031	06449	05059	59884	31880	53115	84469	94868	57967	05811	84514
84177	06757	17613	15582	51506	81435	75011	13006	63395	55041	15866	06589