ANSWER KEY

Sampling Distributions – Homework #6

1. What is the difference between a proportion and a mean?

   LOOK AT CHAPTER 7 OVERVIEW (2 column Summary):

2. A sample is chosen randomly from a population that can be described as normally distributed.
   a) What is the sampling distribution shape, center, and spread compared to the population?

      Shape → normal Center → same as population
      Spread → same as population

   b) If we choose a larger sample, what is the effect on this sampling distribution? What stays the same, what changes and how?

      Shape – same / Normal Center → same
      Spread → smaller / less variability

3. A sample is chosen randomly from a population that is strongly skewed right, …
   a) Describe the sampling distribution for sample mean if the sample size was small.

      Shape → Also skewed / maybe a little less skewed right than population
      Center → same Spread → less variability

   b) As we make the sample size larger, what happens to the expected sampling distribution of sample mean's shape, center, and spread compared to the population's.

      shape → Becomes Normal
      center → same as population
      spread → less variability

4. State police believe that 70% of the drivers traveling on I-90 exceed the speed limit. They plan to set up a radar trap and check the speeds of 80 cars.
   a) What is the expected proportion of speeding cars? $\mu_{\hat{p}} = .70$
   b) What is the standard deviation of the proportion of speeding cars?

      $\sigma_{\hat{p}} = \sqrt{\frac{(.70)(.30)}{80}} = .0512$

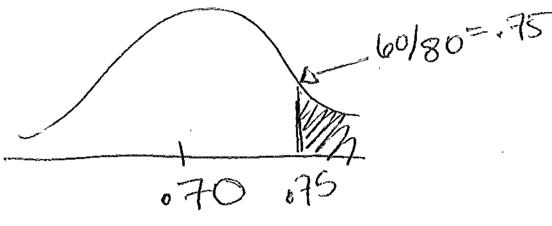
   c) Verify if the appropriate conditions are met. Why is this important? What does it tell us?

      10% condition $10n \le N$ $10(80) \le N$ $800 \le N$
      Def. more than 800 cars on the I90!
      10% condition is important because proves independence.

      $np \ge 10$ $n(1-p) \ge 10$
      $80(.70) \ge 10$ $56 \ge 10$ ✓ $80(.30) \ge 10$ $24 \ge 10$ ✓
      → Therefore, Normal Approx. can be used

   d) If conditions have been met (which they should have), sketch and label a normal distribution with mean and standard deviation from parts *a* and *b*.
   e) What percent (probability) of the time will state police find more than 60 of the drivers speeding?

      $N(.70, .0512)$



$P(\bar{X} \ge .75) = P\left(Z \ge \frac{.75 - .70}{.0512}\right) = P(Z \ge .98)$

$= .1644$

Homework #6 - Question #5 Solution

STATE:

- State what we want to know.

- Find the probability that in a group of 40 attendees, the average distance traveled to a convention is 900 miles or less.

PLAN:

- Check the conditions

- **N ≥ 10n** – N=500, 10n=10(40)=400 500 400. Can use; $\sigma_{\bar{x}} = \sigma/\sqrt{n}$

  n = 40 (large, n>30) – CLT tells use the sampling distribution will be normal.

Do:

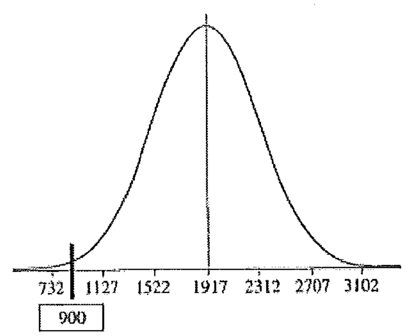
- State the parameters and the sampling distribution model.

- $\mu_{\bar{x}} = \mu = 1917$

  $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2500}{\sqrt{40}} \approx 395.28$$

  therefore, N(1917,395.28)

- Make a picture.

- 

- Write the problem interms of $\bar{x}$.

- $P(\bar{x} \leq 900)$

- Convert to a z-score.
  - $P\left(z \leq \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}\right) = P\left(z \leq \frac{900 - 1917}{395.28}\right) = -2.57$
- Find the resulting probability.
  - $P(z \leq -2.57) \approx .005$

CONCLUDE:

- Discuss the probability in the context of the problem.
  - In a sample of size 40, the probability that the average distance traveled is less than 900 miles is only .005.

Homework #6 - Question #6 Solution

- Given $\mu = 41{,}500, \sigma = 18{,}700, n = 100, \bar{x} = 45{,}510$

- Verify Conditions: - N≥10n, N is the population of a city, it is reasonable to assume it is greater than 1,000. Therefore can use $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.

  - sample size is large (n≥30), therefore by the CLT the sampling distribution will be normal.

- The sampling distribution model is therefore;

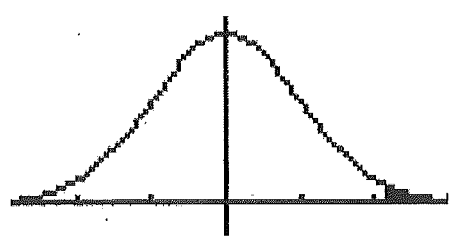
$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N\left(41500, \frac{18700}{\sqrt{100}}\right) = N(41500{,}1870)$$

- Write the problem interms of $\bar{x}$. $P(\bar{x} \geq 45510)$

- Convert to a z-score.

$$P\left(z \geq \frac{\bar{x} - \mu}{\sigma}\right) = P\left(z \geq \frac{45510 - 41500}{1870}\right) = P(z \geq 2.144)$$

- Draw a picture.



- Find the resulting probability. $P(z \geq 2.144) = .016$
- Discuss the probability in the context of the problem.
  - There is a probability of .016 that a sample of 100 will yield a sample mean of $45,510 or higher when the population mean is $41,500.

Homework #6 - Question #7 Solution

- State what we want to know.

- We want to find the probability that in a group of 525 visitors, 70% or more would make a purchase in the gift shop.

- Check conditions:

- **N ≥ 10*n*** - The 525 visitors can be considered a random sample of visitors and it is reasonable to expect the attraction will draw at least 5250 visitors. Can use;

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

***np* ≥ 10 and *n(1-p)* ≥ 10**

np = 525(.65) = 341.25 > 10

n(1-p) = 525(.35) = 183.75 > 10

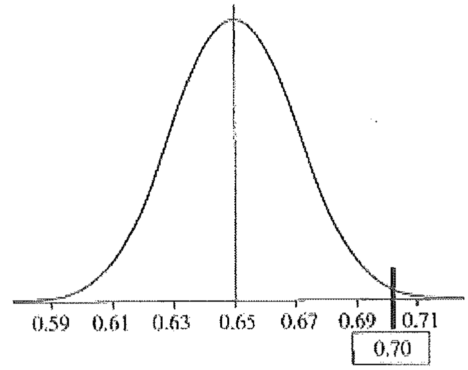
Can use the Normal approximation.

- State the parameters and the sampling distribution model.

- The population proportion is p = .65. The mean of the normal model for is .65 (ie. the mean of the sampling distribution of $\hat{p}$ equals p). The standard deviation is

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.65)(.35)}{525}} \approx .0208 .$$

The model for $\hat{p}$ is N(.65,.0208).



- State the problem in terms of $\hat{p}$.
- $P(\hat{p} \geq .70)$
- Convert to a z-score.
- $P\left(z \geq \frac{.70 - .65}{.0208}\right) = P(z \geq 2.40)$
- Find the resulting probability.
- $P(z \geq 2.40) = .0082$
- Discuss the probability in the context of the question.
- There is a probability of about .0082 that 70% or more visitors will buy something in the gift shop.

Homework #6 - Question #8 Solution

- Verify Conditions:

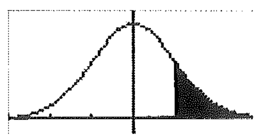
  N ≥ 10*n* 50,000 ≥ 10(100)

  ***np* ≥ 10 and *n(1-p)* ≥ 10**

  (100)(.4) = 40; (100)(.6) = 60

- Therefore, the sampling distribution of is

$$N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N(.4, .049)$$

- Draw a picture. 

- State the problem interms of $\hat{p}$.

$$P(\hat{p} \geq .45)$$

- Convert to a z-score.

$$P\left(z \geq \frac{.45 - .4}{.049}\right) = P(z \geq 1.02)$$

- Find the resulting probability.

$$P(z \geq 1.02) = .1537$$