

Name: ANSWER KEY Period: \_\_\_\_\_ Date: \_\_\_\_\_

Homework #4 – Sample Means

1. **Are college women taller?** The heights of young women follow a normal distribution with mean  $\mu = 64.5$  inches and standard deviation  $\sigma = 2.5$  inches.

a. Calculate the mean and SD of the sampling distribution of  $\bar{x}$  for SRSs of size 15.

$$\mu_{\bar{x}} = 64.5$$

$$\sigma_{\bar{x}} = \frac{2.5}{\sqrt{15}} = .65$$

10% condition

$$10n \leq N$$

$$10(15) \leq N$$

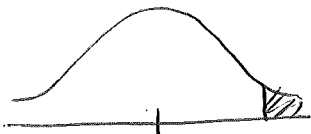
$$150 \leq N$$

b. Interpret the standard deviation from part a.

We expect the average height of 15 women to typically vary by .65 in from the mean of 64.5 in.

c. Find the probability that the mean height of an SRS of 15 young women exceeds 66.5 inches.

$$P(\bar{x} > 66.5) = P\left(Z > \frac{66.5 - 64.5}{.65}\right) = P(Z > 3.10) = .0010$$



64.5 66.5

$N(64.5, .65)$

.1%  $\checkmark$

d. Suppose that the mean height in a sample of  $n = 15$  young women from a local college is  $\bar{x} = 66.5$ . Based on your answer to part c, what would you conclude about the mean height for all young women at this college?

We have convincing evidence that the women at the college have a larger mean height. If the mean were 64.5 in, there is a .1% chance of these women having  $\bar{x} = 66.5$  in.

2. The scores of students on the ACT college entrance examination in a recent year had the normal distribution with mean  $\mu = 20.4$  and standard deviation  $\sigma = 5.8$ .

$$N(20.4, 5.8)$$

a. What is the probability that a randomly chosen student scored 24 or higher on the ACT?

$$P(X > 24) = P\left(Z > \frac{24 - 20.4}{5.8}\right) = P(Z > .62) = .267$$



b. What are the mean and standard deviation of the average ACT score  $\bar{x}$  for an SRS of 30 students?

$$\mu_{\bar{x}} = 20.4$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.8}{\sqrt{30}} = 1.06$$

10% condition  $10n \leq N$   
 $10(30) \leq N$  Def more than  
 $300 \leq N$  300 students  
 taking ACT.

c. What is the probability that the average ACT score of an SRS of 30 students is 24 or higher?

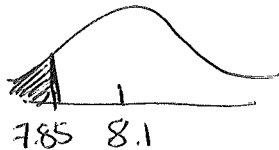
$$N(20.4, 1.06) \rightarrow P(\bar{x} > 24) = P\left(Z > \frac{24 - 20.4}{1.06}\right) = P(Z > 3.40) = .00034$$

3. The distribution of actual weights of 8-ounce chocolate bars produced by a certain machine is normal with mean 8.1 ounces and standard deviation 0.1 ounces. Company managers do not want the weight of a chocolate bar to fall below 7.85 ounces, for fear that consumers will complain.

a. Find the probability that the weight of a randomly selected candy bar is less than 7.85 ounces.

$$N(8.1, 0.1)$$

$$P(X \leq 7.85) = P\left(Z \leq \frac{7.85 - 8.1}{0.1}\right) = P(Z \leq -2.5) = .0062$$



$$.0062$$

Four candy bars are selected at random and their mean weight,  $\bar{x}$ , is computed.

b. Describe the center, shape, and spread of the sampling distribution of  $\bar{x}$ .

Center:  $\mu_{\bar{x}} = 8.1$

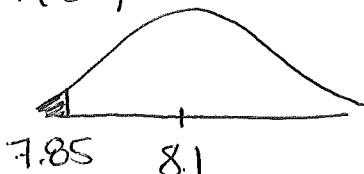
Shape: Normal, since the population is normal

Spread:  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.1}{\sqrt{4}} = .05$

c. Find the probability that the mean weight of the four candy bars is less than 7.85 ounces.

$$N(8.1, .05)$$

$$P(\bar{x} \leq 7.85) = P\left(Z \leq \frac{7.85 - 8.1}{.05}\right) = P(Z \leq -5) = .000000287$$



$$.000000287$$