Lesson 7.2 – The Sampling Distribution of $\hat{p}$

Important ideas:

$\mu_{\hat{p}} = p$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

• 10% condition

LARGE COUNTS

$np \geq 10$

$n(1-p) \geq 10$

$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

Check Your Understanding – Homework # 2

1. **Do you watch online videos?** Suppose that 75% of young adult Internet users (ages 18 to 29) watch online videos. A polling organization contacts an SRS of 1000 young adult Internet users and calculates the proportion $\hat{p}$ in this sample who watch online videos.

a. Identify the mean of the sampling distribution of $\hat{p}$.

N = All internet users that are 18 to 29.

$\mu_{\hat{p}} = p = .75$

b. Calculate and interpret the standard deviation of the sampling distribution of $\hat{p}$. Check that the 10% condition is met. $n = 1000$

$\sigma_{\hat{p}} = \sqrt{\frac{(.75)(.25)}{1000}} = .014$

$10n \leq N \quad 10(1000) \leq N$

$10{,}000 \leq N$

Definately more than 10,000 (18 to 29 yr old) internet users.

c. Is the sampling distribution of $\hat{p}$ approximately Normal? Check that the Large Counts condition is met.

$n \cdot p \geq 10$

$1000(.75) \geq 10$

$750 \geq 10$ ✓

$n(1-p) \geq 10$

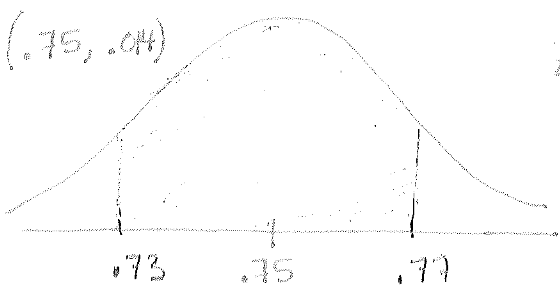
$1000(.25) \geq 10$

$250 \geq 10$ ✓

Yes, Normal approximation can be used.

d. Find the probability that the random sample of 1000 young adults will give a result within 2 percentage points of the true value. $P(.73 < \hat{p} < .77) =$

N(.75, .014)



$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

$z = \frac{.73 - .75}{.014} = -1.43$

$z = \frac{.77 - .75}{.014} = 1.43$

normalcdf(.73, .77, .75, .014) = .8557

e. If the sample size were 9000 rather than 1000, how would this change the sampling distribution of $\hat{p}$?

Shape stay the same → Normal

Center stay the same → .75 = p

Spread (standard deviation → decreases) Less variability

2. **Who goes to church?** The Gallup Poll asked a random sample of 1785 adults whether they attended church during the past week. Let $\hat{p}$ be the proportion of people in the sample who attended church. A newspaper report claims that 40% of all U.S. adults went to church last week. Suppose this claim is true.

n

$p = .40$

N = All U.S. adults

(a) Calculate the mean and standard deviation of the sampling distribution of $\hat{p}$. Interpret the standard deviation.

$\mu_{\hat{p}} = .40$

10% condition

$10n \le N$

$10(1785) \le N$

$17{,}850 \le N$

There are definately more than 17,850 U.S. Adults.

$\sigma_{\hat{p}} = \sqrt{\frac{(.4)(.6)}{1785}} = .012$

(b) Justify that the sampling distribution of $\hat{p}$ is approximately normal.

LARGE COUNTS

$n \cdot p \ge 10$

$(1785)(.40) \ge 10$

$714 \ge 10$ ✓

$n(1-p) \ge 10$

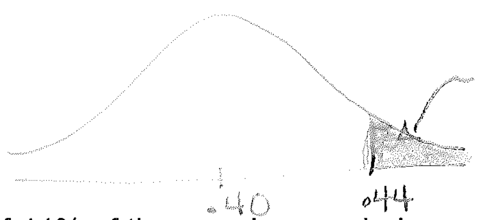
$1785(.60) \ge 10$

$1071 \ge 10$ ✓

(c) Calculate the probability that at least 44% of the people in the sample attended church.

$N(.40, .012)$

$z = \frac{.44 - .40}{.012} = 3.33$



normalcdf(.44, 1E99, .40, .012) = .00043

(d) If 44% of the people sampled were found to attend church, would this be convincing evidence that the newspaper was incorrect?

Yes! Because there is A .00043 chance of it happening. (Anything less than 5% is alarming = STATISTICALLY SIGNIFICANT!)