Homework #3

Transforming Random Variables Homework

REMINDER:
Std. Deviation = $\sqrt{\text{var}\,iance}$
$\sigma = \sqrt{\sigma^2}$

1) Random variable X has a mean of 6.2 and a standard deviation of 3.1. $\mu_x = 6.2$ $\sigma_x = 3.1$

   a. Find the new mean and standard deviation if we multiply by 3

   $\mu_{3x} = 3(6.2) = \boxed{18.6}$ $\sigma_{3x} = 3(3.1) = \boxed{9.3}$

   b. Find the new mean and standard deviation if we subtract 10

   $\mu_{x-10} = 6.2 - 10 = \boxed{-3.8}$ $\sigma_{x-10} = \boxed{3.1}$

   c. Find the new mean and standard deviation if we multiply by 5 and add 10.

   $\mu_{5x+10} = 5(6.2) + 10 = \boxed{41}$ $\sigma_{5x+10} = 5(3.1) = \boxed{15.5}$

2) Random variable Y has a mean of 3.4 and a standard deviation of 1.4. → $\mu_y = 3.4$ $\sigma_y = 1.4$

   a. Find the mean and standard deviation of X + Y

   $\mu_{x+y} = 6.2 + 3.4 = \boxed{9.6}$ $\sigma_{x+y} = \sqrt{(3.1)^2 + (1.4)^2} = \boxed{3.4}$

   b. Find the mean and standard deviation of X – Y

   $\mu_{x-y} = 6.2 - 3.4 = \boxed{2.8}$ $\sigma_{x-y} = \boxed{3.4}$

   c. Find the mean and standard deviation of 2X + 3Y

   $\mu_{2x+3y} = 2(6.2) + 3(3.4) = \boxed{22.6}$ $\sigma_{2x+3y} = \sqrt{(2 \cdot 3.1)^2 + (3 \cdot 1.4)^2} = \boxed{7.5}$

   d. Find the mean and standard deviation of 3X + Y – 4

   $\mu_{3x+Y-4} = 3(6.2) + (3.4 - 4)$
   $18.6 + (-.6)$
   $\boxed{18}$

   $\sigma = \sqrt{(3 \cdot 3.1)^2 + (1.4)^2}$
   $86.49 + 1.96$
   $= \sqrt{88.45} = \boxed{9.4}$

3) Lets play a game! A single dice is rolled and the following occurs:

- Roll a 6, get 40 points
- Roll a 4 or 5, get 10 points
- Roll a 1, 2, or 3, get 0 points X = POINTS

$\boxed{\mu_x = 10}$

a. Write the probability distribution

L1	X	0	10	40
L2	P(X)	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

$\mu_x = 0(3/6) + 10(2/6) + 40(1/6) = 10$

$\sigma_x = \sqrt{(0-10)^2(\frac{3}{6}) + (10-10)^2(\frac{2}{6}) + (40-10)^2(\frac{1}{6})}$

b. Find the mean and standard deviation of the game

expected value

$(x-\mu)^2(p)$ $\boxed{\sigma_x = 14.14}$

c. Suppose the points are doubled. Find the new mean and standard deviation

$\mu_{2x} = 2(10)$
$= 20$

$\sigma_{2x} = 2(14.14)$
$= 28.28$

d. Supposed the game is played twice (independently). What is the mean and standard deviation of this?

$\mu_{x+x} = 10 + 10$
$= 20$

$\sigma_{x+x} = \sqrt{(14.14)^2 + (14.14)} = \boxed{20}$

More examples:

1. Suppose that *X* is a random variable with $\mu_X = 10$ and $\sigma_X = 2$. Find the following:

   a. $\mu_{5X} = 5(10) = 50$

   b. $\mu_{X-3} = 10 - 3 = 7$

   c. $\mu_{5X-3} = 5(10) - 3 = 47$

   d. $\sigma_{5X} = 5(2) = 10$

   e. $\sigma_{(X-3)} = 2$

2. Suppose that *X* is the random variable from above, and that *Y* is independent from *X* with $\mu_Y = 15$ and $\sigma_Y = 3$. Find the following:

   a. $\mu_{X-Y} = 10 - 15 = -5$

   b. $\mu_{X+3Y} = 10 + 3(15) = 55$

   c. $\sigma_{X+Y} = 3.6$

   d. $\sigma_{X-Y} = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.6$

   e. $\sigma_{5X-2Y} = \sqrt{(5 \cdot 2)^2 + (2 \cdot 3)^2} = 11.66$

3. Suppose the mean SAT verbal score is 425 with standard deviation 100, while the mean SAT math score is 475 with standard deviation 100. What can be said about the mean and standard deviation of the combined math and verbal score?

   $\mu_{V+M} = 425 + 475 = 900$

   $\sigma_{V+M} = \sqrt{100^2 + 100^2} = \sqrt{20000} \approx 141.42$

4. In a large introductory statistics class, the distribution of *X* = raw scores on a test was approximately normally distributed with a mean of 17.2 and a standard deviation of 3.8. The professor decides to scale the scores by multiplying the raw scores by 4 and adding 10.

   a. Define the variable *Y* to be the scaled score of a randomly selected student from this class. Find the mean and standard deviation of *Y*.

   $\mu_{4X+10} = 4(17.2) + 10 = 78.8$

   $\sigma_{4X+10} = 4(3.8) = 15.2$

   b. What is the probability that a randomly selected student has a scaled test score of at least 90?

   $P(X \geq 90) = \text{normalcdf}(90, 1E99, 78.8, 15.2) =$

5. Lebron James and Michael Jordan would like to compare their points per game. Michael's points are normally distributed with a mean of 30.1 PTS and a standard deviation of 6. Lebron's points are also normally distributed with a mean of 26.5 PTS and a standard deviation of 5.

   $\mu_{M-L} = 30.1 - 26.5 = 3.6$ $\sigma = \sqrt{6^2 + 5^2} = \sqrt{36 + 25} = \sqrt{61} \approx 7.8$

   c. Find the mean and standard deviation of the differences their scores.

   d. Find the probability that Lebron will have a higher PTS than Michael if they were to play one another.

   $P(X < 0) = \text{normalcdf}(-1E99, 0, 3.6, 7.8) =$