

A₁

4. Two antibiotics are available as treatment for a common ear infection in children.

- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs \$50.
- Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs \$80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?

$$\begin{aligned}P(\text{cure}) &= P(\text{A cures it}) + P(\text{B cures it}) - P(\text{A didn't cure it}) \\P(\text{cure}) &= .6 + (.9)(.4) \\&= .6 + .36 \\P(\text{cure}) &= .96\end{aligned}$$

If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

$$\begin{aligned}P(\text{cure}) &= P(\text{B cures it}) + P(\text{A cures it}) - P(\text{B didn't cure it}) \\&= .9 + (.6)(.1) \\&= .9 + .06 \\P(\text{cure}) &= .96\end{aligned}$$

(b) Compute the expected cost per child when plan I is used for treatment.

$$\begin{aligned}
 \text{expected cost} &= \text{cost A} + \text{cost B} \cdot p(\text{A doesn't cure it}) \\
 &= \$50 + \$80(.4) \\
 &= \$50 + \$32 \\
 &= \$82
 \end{aligned}$$

Compute the expected cost per child when plan II is used for treatment.

$$\begin{aligned}
 \text{expected cost} &= \text{cost B} + \text{cost A} \cdot p(\text{B doesn't cure it}) \\
 &= \$80 + \$50(.1) \\
 &= \$80 + \$5 \\
 &= \$85
 \end{aligned}$$

(c) Based on the results in parts (a) and (b), which plan would you recommend?

Explain your recommendation.

I would recommend plan I. The probability that the child will be cured is the same, so that does not effect the choice. However, on average, plan I will cost less than plan II. Each child will cost on average \$82 on plan I, while costing \$85 on plan II. Since their insurance companies normally end up paying for the medication, they would want to recommend the plan that would cost them the least per child on average.